MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXII. SUPPLEMENTARY NUMBER. NO. 9

The Society has suffered a heavy loss in the death of the Treasurer, Col. E. H. Grove-Hills, which occurred on October 2.

On the Determination of the Colour-equivalent of a Star, with special reference to the Effective Wave-length and its Relation to Spectral Class: A Review of the Different Methods. By Knut Lundmark, Ph.D., and W. J. Luyten, Ph.D.

As colour-equivalent of a star may be taken every well-measurable quantity which in a simple way depends on the colour itself. From its nature the latter can, strictly speaking, be determined only by direct estimates with the naked eye, and is as such not very well suited to measuring purposes.

Colours of the stars were mentioned by the ancients (1), were known to Ptolemy, and have in recent years been determined by visual estimates, sometimes with the aid of small telescopes, by Santini, Schmidt, Krueger, Osthoff, Lau, Müller, and Kempf, and others. In this way the colour of a star can be found only as a relative quantity, and it has to be referred to some standard system in somewhat the same way that the apparent magnitude of a star is referred to the Harvard system (2).

As a theoretical scale of colours there might be used a sequence of colour photographs of bright stars obtained by the Lippmann method.

Although these visual estimates can be very accurate (Osthoff), they are restricted to bright stars; for this reason, and also because modern astronomy concerns itself with measurable quantities, various other methods for determining colour-equivalents have been developed. Among the principal ones may be mentioned:—

I. The colour-index; *i.e.* the difference in intensity, expressed in

magnitude, between two different parts of the spectrum; or, in mathematical symbols,

$$\text{C.I.} = 2{\cdot}5 \log \left[\int_{\lambda_3}^{\lambda_4} \Phi(\lambda) d\lambda - \int_{\lambda_1}^{\lambda_2} \Phi(\lambda) d\lambda \right] \quad . \quad . \quad (1)$$

where $\Phi(\lambda)$ is the observed spectral intensity at wave-length λ.

The different colour-indices enumerated below are then found by varying both the function Φ and the limits of the integrals. From this point of view it should be possible, from spectral-photometric work, to *compute* colour-indices.

In practice there have been used:—

A. The colour-index proper, the difference photographic – visual magnitude;
B. The difference radiative – visual magnitude; and
C. The difference radiative – photographic magnitude, as developed by Coblentz (3) and Burns (4);
D. Ultraviolet-photographic magnitude introduced by Seares (5);
E. The difference between photoelectric magnitudes obtained without and with colour filter (6);
F. The colour-index obtained by the use of Nordmann's photomètre hétérochrome (7).

II. Seares's method of exposure ratios (8).

III. King's method of taking simultaneous photographs of a star through a yellow and a blue filter, each covering one half of the objective (9).

IV. Tamm's extrafocal photography through UV lenses (10).

V. Effective wave-lengths.

Undoubtedly other equivalents will be used in future. Care should be taken, however, when developing any new equivalent, not to introduce quantities too sensitive to such instrumental errors as achromatism and photographic effects, or quantities too hard to measure, such as $\lambda_{\min}$, to be explained later.

We shall now try to give a more detailed description of the methods mentioned here, and compare the pros and cons of each method.

I. Determination of colour-index rests on the fundamental scales for all of the different magnitudes used. For the photographic magnitudes there is the well-established photographic North Polar Sequence, as determined and confirmed by the co-operation of Harvard, Mount Wilson, and Greenwich. For the brighter visual magnitudes we have the Harvard system, which seems also fairly well established, although there is still the systematic scale difference Harvard – Parkhurst that needs explanation. A North Polar Sequence of photovisual magnitudes for fainter stars has been determined by Seares (11), but this can only be regarded as an extension of the visual series if no systematic difference between the two exists. The facts are, however, that the two scales coincide only at 6th and 12th magnitudes, and that the difference $H_{vis} - S_{photovis}$ depends on the colour. What this finally comes to, is that in the colour-index are involved systematic errors depending on the colour; or, in other words, what we actually determine is a function of the colour and some unknown parameters. Fortunately, these unknown

quantities do not play an important part, and do not sensibly diminish the value of a colour-index determined in this way. Von Zeipel and Lindgren (12) have determined photovisual magnitudes on Seares's scale and visual magnitudes extrapolated from the Potsdam system. From a comparison of these values for 218 stars they find a mean systematic correction in the sense

$$m_{\text{vis}} - m_{\text{p.v.}} = -0^{\text{m}}\cdot01 \pm 0^{\text{m}}\cdot005,*$$

whereas Seares (13) finds for the same difference

$$+0^{\text{m}}\cdot19 \pm 0^{\text{m}}\cdot02 \text{ from } 12 \text{ stars.}$$

Von Zeipel concludes from this that his visual magnitudes are in error. This is probably the case, but still the connection between the visual and photovisual scales needs further attention.

Regarding the colour-indices mentioned under B and C, these, although they are extremely important for individual stars, cannot as yet be used in general considerations, as they are at present, unfortunately, restricted to bright stars. Method D is only in the development stage, and not much can be said of it.

The same holds for E, which has been applied by Guthnick and Hügeler to Nova Aquilæ, and which seems to give results of extremely high accuracy. A comparison with other colour equivalents shows a linear relation in most cases. It is very remarkable, on the other hand, that this colour-index, F_1, decreases so considerably from M*a* to M*d*, but increases towards N.

Nordmann and Le Morvan have determined colour-indices by measuring visually through red and blue screens. The range in colour-index obtained in this way is very close to that determined by the difference photographic – visual.

In deriving the actual magnitudes from the images on photographic plates we have the advantage that either the diameter or the blackness can be used. Both quantities are suited to refined measurement, especially in the last case, when a "Schraffier Kassette" or intrafocal images in conjunction with a microphotometer are used. The personal element in measuring can be entirely eliminated by using the self-recording microphotometer of Koch, or its recent improvement by Moll (14).

Among the disadvantages of the colour-index determinations may be mentioned:

(*a*) Dyeing of plates may introduce systematic errors, or possibly a non-uniform sensitivity of the plates (15).

(*b*) Corrections have to be applied corresponding to distance from centre and these may contain uncertainties.

(*c*) If an auxiliary scale is used, and magnitudes are determined by estimates, as in the Argelander method, trouble will arise from the definition of the images on the plates. This can be greatly overcome by using different scales corresponding to varying definitions. There also seems to be a change in personal equation with the time (15).

Among the more extensive determinations of colour-index, apart from the standard-sequence work, may be mentioned those of Park-

* By a typographical error given as $0^{\text{m}}\cdot05$ by von Zeipel.

hurst (16), Schwarzschild (17), King (18), Shapley (19), von Zeipel and Lindgren (12).

II. Seares's method.

Advantages:—1. Every part of the field can be used, as the shape of the images will not greatly influence the accuracy; and

2. Measuring is not necessary; estimates will do, especially as a large number of images are secured on the same plate.

Disadvantages:—1. The connection between exposure ratios and colour is not known *a priori*, and has to be determined in the course of the work (from known colour-indices);

2. It is tacitly assumed that the filters and colour-sensitiveness of the plates are invariable (this applies, of course, to all methods using dyed plates and colour filters);

3. The variation in the transparency of the air will undoubtedly influence the results. (This will, however, not be of great importance for the observatories on the Pacific Coast); and

4. The colour derived by this method is dependent on the intensity of the image (photographic Purkinje effect).

As a knowledge of standard sequences is not necessary, this method of determining the colour-index looks very promising for the future, and it can at the same time afford a valuable check on the colour-indices obtained by method I. (20).

III. and IV. Not much can as yet be said of these methods, as we have only a preliminary knowledge of them. In method IV. the differently-coloured stars give a varying intensity of the outside ring as compared with the central dot. From the appearance of records for the different spectral types (judging from prints and information kindly communicated by Mr. Tamm) satisfactory results may be predicted. The method has the advantage that an achromatic lens is not required, a simple lens being, in fact, preferable.

V. Effective wave-lengths.

The term effective wave-length was first applied by Comstock (21) to his visual determinations of the wave-length for the maximum intensity in short objective-grating spectra. For the different spectral classes the differences in effective wave-length are very small in the visual region, and determinations of this quantity are therefore made mostly by photography.

A grating of equal bars and spaces is placed in front of the objective, as was first done by the Brothers Henry (22) and by Schwarzschild (23), and later applied to the determination of colour equivalents by Hertzsprung (24) and Bergstrand (25). More recently measures of effective wave-length have been made by Lindblad (26), Lundmark (27), Wolf (28), Davidson and Martin (29), Balanovsky (30), and others. There is no need here to go into the theory of the objective grating, as this can be found in the earlier of the papers referred to.

If $\Phi(\lambda)$ is the spectral intensity for the wave-length λ, then the effective wave-length can be defined by the expression

$$\lambda_{\text{eff}} = \frac{\int \Phi(\lambda)\lambda d\lambda}{\int \Phi(\lambda) d\lambda} . \quad . \quad . \quad . \quad . \quad (2)$$

both integrals extending over all wave-lengths acting on the photographic plate. As this quantity is intimately related to the effective temperature of a star and thus also to the colour, the λ_{eff} as measured, even by using small instruments, gives a valuable check on the colour-index determined by other methods. Used in connection with powerful instruments λ_{eff} will afford material for an extended knowledge of the colours of faint stars.

The construction of a coarse grating does not introduce such delicate technical problems as are met in the manufacture of colour screens. Besides, an objective grating may be more easily kept in order and controlled than a colour screen.

If always used in the same position relative to the objective, no trouble will arise, even if the grating is slightly tilted relatively to the optical axis, as this introduces only a small systematic error. As is the case with Seares's, King's, Tamm's, and Nordmann's methods, work can be done for λ_{eff} independently of a photometric scale. Neither is any dyeing of the plates needed, with accompanying uncertainties, and almost any kind of high-speed commercial plates can be used to advantage. Leaving out of consideration the correction of λ_{eff} for the relative strength of the image, the reduction of the measures is simple and does not involve much labour.

One of the principal advantages with the λ_{eff} method is perhaps its usefulness for objects of measurable extension on the plate, such as nebulæ, planets, and comets. The use of other methods, except Seares's, and perhaps Guthnick's, seems to present difficulties in these cases.

On the other hand, it must be admitted that the method is of rather idiosyncratic behaviour, and, if not carefully used, it certainly contains several disadvantages. A number of these have come within the experience of the authors in preliminary work undertaken with the grating, and on account of the growing interest in this subject a brief discussion of some of them is given here.

Every observer of λ_{eff} has met with the so-called photographic Purkinje effect, *i.e.* the dependence of λ_{eff} on the intensity of the images on the plate. (This follows immediately from formula (2).) To overcome this, the material has generally been reduced to an arbitrarily selected value of the diameter of the central image. Now the effect depends also on the spectral class, and the correction can thus be obtained only by successive approximations, as λ_{eff} is unknown. The sometimes complicated forms of the empirical reduction curves used in correcting for this effect show how much care should be taken in order to get an accurate correction for this important source of error. For some instruments the effect has been found to be so large that the measures cannot properly be used for obtaining knowledge of λ_{eff} (31). On the other hand, this effect appears to be very small, if sensible at all, in the determinations of λ_{eff} for surface objects with the Upsala 6-inch twin telescope; at least so long as large intensities are not used.

The effective wave-lengths as measured are in a high degree dependent on the optical properties of the instruments used. Regarding refractors, nothing can be said before a thorough test has been made. The system of λ_{eff} depends entirely on achromatism. Thus it was

found in Upsala that changes in the optical system (the middle lenses of the triplets were slightly readjusted to obtain good focus over as large a field as possible) changed the λ_{eff} so much that another scale had to be established. Reflectors should be well adapted for work of this kind. Their behaviour has, however, not quite come up to theoretical expectations. A comparison between the results of Bergstrand, Wolf, and Greenwich is, in this respect, very instructive. Rather obscure is the apparent relation of the system of λ_{eff} to a resilvering of the mirror, as found by Wolf, and described in his paper.

A connection between λ_{eff} and changes in focal length, arising from temperature, has been suspected.

Rosenberg (32) has investigated this problem and found, by using a grating of high precision, that changes of the focal length as small as $\frac{1}{1680}$ could be detected in the measured effective wave-lengths. Of still greater importance is his result that small focal changes influence very considerably the shape of the reduction curves for the photographic Purkinje effect.

Other optical effects are the distortion of the images by distance from centre and by atmospheric dispersion. These can be made less serious by reducing the aperture of the objective, by avoiding too long exposures, or by working at small zenith distances. The Greenwich observers have measured their λ_{eff} for stars with distorted images, as they found those no less fit for accurate measurement. They do not state, however, whether there exists any systematic difference between the λ_{eff} for those images and for images near the centre of the plate.

In an instrument used by Lindblad and one of the authors a rotational error was found, *i.e.* λ_{eff} changes when the grating is rotated in front of the objective.

Sometimes a chromatic irregularity of the lens is shown, which acts in much the same way as the atmospheric dispersion. The most natural explanation is that the lens acts as a prism, and in the resulting spectra there is dispersion in two directions.

The state of the atmosphere is of some importance for work of this kind. Messrs. Vallin and Lundmark thus found a fogging effect for λ_{eff} at Upsala. When comparing λ_{eff} obtained through a clear sky and low humidity with those obtained through a foggy sky and high humidity, a mean difference of 30 Å was found in the sense that a fog increases λ_{eff}. Davidson and Martin (29) also mention an influence of the same sort. They do not state, however, whether this effect is systematic and in the same direction.

The influence of chemical fog or other photographic effects has not yet been investigated; neither has any comparison been made between different kinds of plates. Preliminary work of Vallin, as given in Lindblad's paper, shows the importance of a proper selection of plates. In the measurement of the plates a personal element is introduced, as the short spectra generally are not symmetrical and their appearance varies with the spectral class. When measuring λ_{eff} of spirals and clusters Lindblad and Lundmark found very good agreement, taking into account the difficulty of measuring objects of this kind. From a great number of measures there resulted a mean difference between

the two observers of 1 Å, which is quite negligible. Comparison of measures of stars has shown very little systematic difference between these two observers for all spectral classes except B, where Lindblad measures λ_{eff} decidedly smaller. Another comparison between two observers is given in figure 1 where, among others, are shown the results of Bergstrand and Lindblad, both using the same instrument and the same kind of plates.

No clear definition exists in the literature of the subject as to whether the λ_{eff} is the wave-length found by bisecting the blackened area or by setting on the "centre of gravity," then giving λ_{eff} as defined by (2). It is evident that in practice the last way had been followed, and although a measure of this kind may seem only vaguely defined, accuracy in measuring can certainly be obtained after some experience. Among the disadvantages of the λ_{eff} method may also be mentioned that in dense regions the position of the short spectra can be influenced by photographic effects, sometimes called photographic repulsion (investigated by F. E. Ross) (33). This applies also to the other colour-equivalent methods, but in less degree, because, when dealing with effective wave-lengths, we have at least three times as many images on the plate. Compared with the colour-index method, λ_{eff} is relatively somewhat inefficient; for it has been found that when using the grating, exposures must be increased at least ten to fifteen times, whereas the colour screen increases the exposures only six times. Moreover, in the latter case, underexposed images are still measurable, which is not true for the short spectra.

It is very doubtful whether λ_{eff} has any real significance for classifying purposes when applied to objects with bright emission lines. Thus the extraordinarily low value of 3940 Å for the Ring Nebula, as found by Lundmark and Lindblad, must be ascribed to the presence of the bright doublet at 3727 Å. The spirals gave wave-lengths corresponding to the spectral classes G–K, in close accordance with the results from actual spectral classification. For the planetaries a mean effective wave-length of 4130 Å was found, corresponding to spectral class B–F. It seems thus possible with this instrument to separate the spirals from the planetaries, which is sometimes impossible to do from considerations of structure alone. In a recent paper Balanovsky finds for 16 objects, probably spirals, a mean effective wave-length of 4420 Å (in his system corresponding to G), in good agreement with Lindblad and Lundmark. On the other hand, 4 of his planetaries give in the mean 4760 Å, which abnormal value seems to be hard to explain for the present, even if we may assume that N_1 and N_2 have contributed to the recorded spectrum. Although he thus gets λ_{eff} higher for planetaries than for spirals, which is contrary to the Upsala results, still his system can be used for distinguishing between these two kinds of nebulæ.

The close dependence of λ_{eff} on the optical system and the several other effects mentioned are the reason why no two results of different observers are directly comparable. It is thus to be regretted that no standard sequence has been established. It may be suggested to perform this by laboratory experiments with monochromatic light.

However, until this has been accomplished, astronomers should be careful in their interpretation of observations of effective wave-lengths, and should not use them without reduction, as has in some cases been done for the derivation of colour-equivalents. Likewise, we are of the

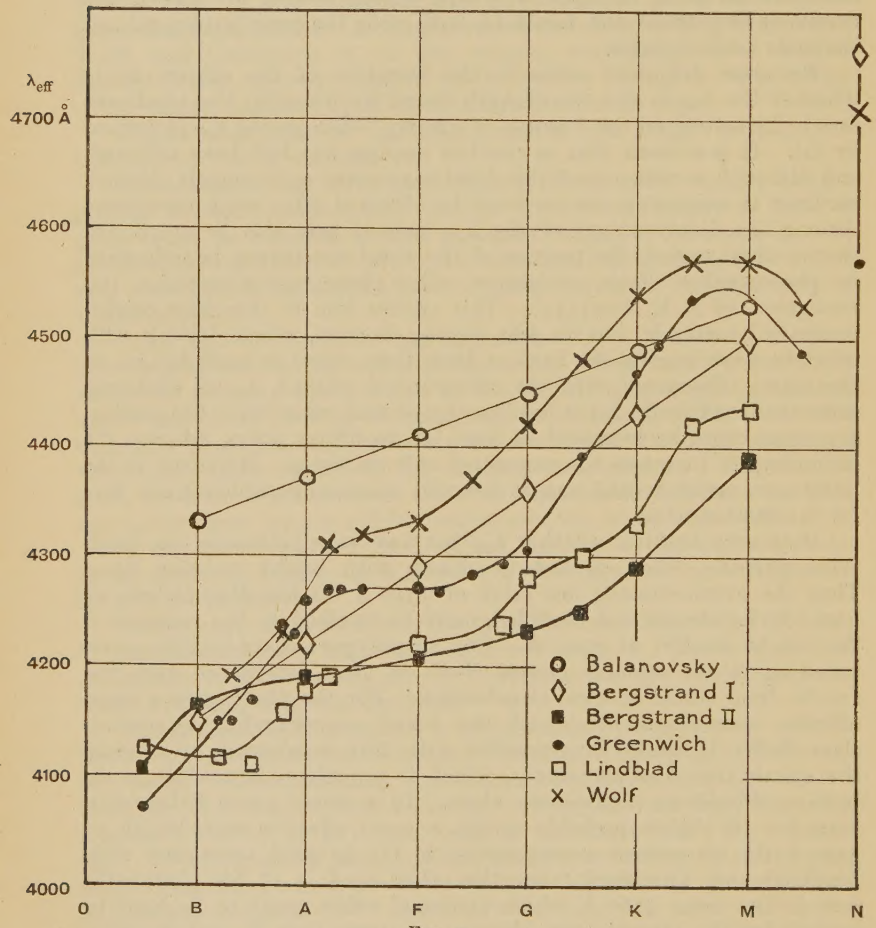

FIG. 1.

opinion that it is not justifiable to use the value of the effective wave-lengths for a certain spectral class determined by *another* observer, with *another* instrument, on *other* plates, as a point of reference for one's own measures, which procedure has been followed by two investigators (34).

In spite, however, of all the systematic errors to which a determination of λ_{eff} is subject, there is, unquestionably, a relation between λ_{eff} and spectral class. On the main features of this relation there is a

general agreement among observers, as may be seen from fig. 1, where the effective wave-length measures of Balanovsky, Bergstrand I., Bergstrand II., Greenwich, Lindblad, and Wolf are plotted against spectral class. It appears from this that the increase of λ_{eff} is rather rapid between B and A, is slow (for some observers almost zero) from A to F,* and large again from these onward, till at M it goes down toward Mb, c, and up toward N.

In their paper Davidson and Martin publish an extensive diagram giving the relation between their individual λ_{eff} and spectral class; from this it will be seen that at G5, *e.g.* the measured effective wave-lengths cover the entire range of the mean λ_{eff} from A to K5. When it comes to discussing the possible causes of this unexpected effect, we are, at present, able to arrive at definite conclusions respecting the error in spectral classification. For, apart from the Harvard spectra used by Davidson and Martin, we have for the stars in this part of the sky another determination of the spectral classes by ten Bruggen Cate (35). As his paper deals with a comparatively small number of stars, and great care was taken in the classification, we can give this determination a weight at least equal to that of the New Henry Draper Catalogue. A comparison between the two different classifications will then give us an idea of the error to be expected in a single determination of spectral class. If Ao is denoted by 0 and Ko by 30, then the relation between Harvard and ten Bruggen Cate can be written

$$B = 0{\cdot}89\ H + 0{\cdot}4.$$

The deviations from this straight line lead to the following mean errors in the classification:

For Ao	± 0·7	
Fo	2·2	
F5	3·1	unit: one spectral
Go	3·4	subdivision
G5	2·8	
Ko	2·2.	

It is clear from this that errors in spectral classification alone cannot be responsible for the large discordance in λ_{eff} for G5 – K, as the mean errors decrease where the discordances increase. In fig. 2 are plotted the Greenwich λ_{eff} against ten Bruggen Cate's spectra; each star is represented by an ellipse with axes equal to the mean errors in the two co-ordinates.

Another estimate of the allowable error in spectral classification is found from a comparison between Harvard and Mount Wilson spectra for 489 stars (between 0 and 8 hours of R.A., and between Fo and M, taken from *Mount Wilson Contributions*, No. 199).

The spectral classes were again denoted by numbers and a rigorous solution by least squares was made in the following way: Let each star be represented by a point whose x equals the Harvard spectrum, and

* Due to the well-known ultraviolet absorption in Class A stars.

whose y equals the Mount Wilson spectrum; then the problem is to lay a straight line through these points in such a way that the sum of

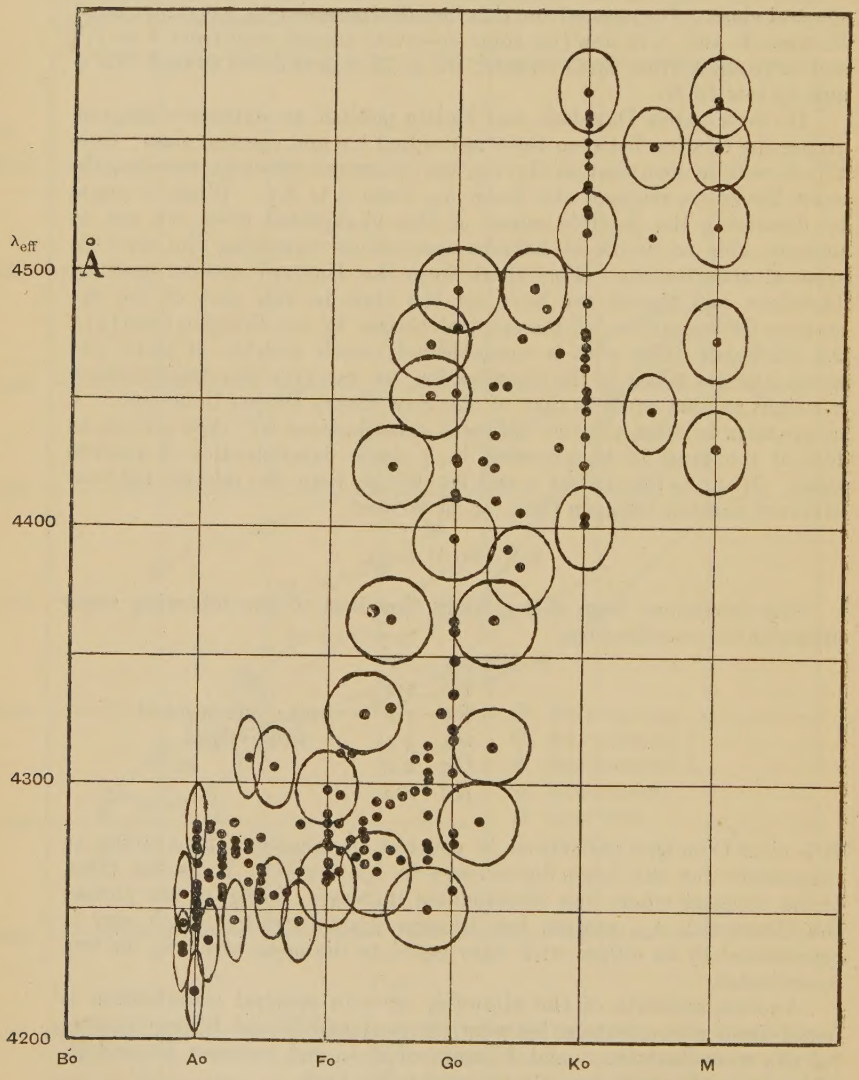

FIG. 2.

the squares of the perpendicular distances of these points from the line is a minimum. Write the equation of the line as follows: $L \equiv -x \sin \alpha + y \cos \alpha - l = 0$ (where α is the angle between the line and the axis of

x); then $D(x_1y_1) = L(x_1y_1)$; accordingly, $\Sigma L^2(x_1y_1) = \text{Min.}$ After differentiation and substitution we find for a and l

$$\tan 2a = \frac{2\Sigma xy - \frac{2}{n}\Sigma x\Sigma y}{\Sigma x^2 - \Sigma y^2 + \frac{1}{n}\Sigma y\Sigma y - \frac{1}{n}\Sigma x\Sigma x} \quad \text{and} \quad l = \frac{\cos a \,.\, \Sigma y - \sin a \Sigma x}{n}$$

where n is the number of stars. This derivation, however, only holds when the mean errors in both co-ordinates are equal; to obtain this the ordinates were increased corresponding to giving double weight to Mount Wilson.

Solution of the equations then yields

$$W = 1{\cdot}15(\pm{\cdot}17)H - 1{\cdot}6(\pm 2{\cdot}0)$$

on a scale where Fo = 0 and Ma = 30.

The mean errors in spectral classification then turn out:

$$\pm 2{\cdot}0 \text{ for Harvard, } \pm 1{\cdot}4 \text{ for Mount Wilson,}$$

these being of the same order of magnitude as previously found.

Another useful parameter to indicate errors in spectral classification is the correlation constant. This constant r is in the case Harvard-Mount Wilson

$$r = 0{\cdot}932.$$

This is relatively low; for Charlier (36) found as high a value as 0·958 for the correlation constant between Harvard spectra and Parkhurst's colour-indices.

The difference Mount Wilson-Harvard was previously investigated by Adams and Joy (37), who found $-1{\cdot}6$ spectral subdivisions. On the basis of a more comprehensive inquiry they derived later on (38) a difference of opposite sign, varying irregularly with the spectral class itself. It may also be mentioned that they found a pronounced difference in the values H – W for giants and dwarfs, in the sense that Mount Wilson classifies the dwarfs comparatively redder than it does the giants (by approximately two spectral subdivisions). We shall refer to this fact later on.

If we now consider another possible cause of the discrepancy in the λ_{eff} for late-type stars, bearing on the irregularities in the spectra (and accordingly in the λ_{eft}) of the stars, depending on the giant or dwarf character (pointed out by Seares and by Lindblad), we come to the conclusions that this cannot produce such a marked effect on the range. In agreement with present data we assume that among the Ko stars the dwarfs outnumber the giants *in space* by 100 to 1; further, that their respective mean absolute magnitudes are $+6^m{\cdot}0$ and $+0^m{\cdot}8$, both with a Gaussian distribution. After making use of Kapteyn and van Rhijn's density law (39) we find that at apparent magnitude $8^m{\cdot}5$ (lower than the mean of the Greenwich stars) the dwarfs form only 30 per cent. of the total number. This would accordingly cause a spread on the *blue* side of the maximum, whereas Davidson and Martin's figure shows most of the dispersion on the *red* side.

As the evidence we have regarding the errors in the measures of effective wave-length (for one instrument only) does not account for discrepancies of the size of those indicated in the plot, we come to the conclusion that at present individual effective wave-lengths cannot serve to predict spectral class with certainty.

Whether this is due to the several causes mentioned above or to real differences in λ_{eff} even among giants of the same spectral class, we cannot yet say, but the facts seem to point largely towards the latter possibility.

An inspection of fig. 1 will strengthen our conclusion that individual λ_{eff} cannot serve for the prediction of spectral class. It will be seen that between A and G λ_{eff} is nearly independent of the spectral class, whereas between G and M the uncertainty is too large, or the curve reverses (Wolf), and the only safe predictions that can be made are for the classes O and N (*cf.* Wolf's list in *A.N.*, 5092, where his estimates differ materially from the spectral classification, *e.g.* as the star called B8 by him is classed F8 by Harvard). We can therefore say that the relation between spectral class and effective wave-length is not in the nature of a one to one correspondence. The most that can be said is that upon the spectral class a rough estimate of λ_{eff} may be based, but that the direction of inference may not be reversed.

It is clear from the foregoing considerations that we must receive with much scepticism any method of deriving the absolute magnitude from a combination of effective wave-length and another colour equivalent. This is especially so, when for this second colour equivalent the minimum effective wave-length is used, as this asymmetrical quantity is subject to a large personal element in measuring.

Such a method was developed by Lindblad (40), and although from a theoretical point of view a difference in λ_{min} should exist between giants and dwarfs, it is very doubtful whether this can be detected even in the refined measures of Lindblad. For with his arrangement of the grating (wires north and south, and therefore length of spectra in Right Ascension) any error in guiding will affect λ_{min} much more than it will λ_{eff}. A more serious objection that can be raised against the principle of using λ_{min} and λ_{eff} together for prediction of luminosity is the high correlation that exists between them. From the 64 stars measured by Lindblad (41) the correlation constant r is computed to be

$$r = 0{\cdot}969,$$

from which we draw the conclusion that the correlation is absolute and that his measures are extremely accurate.

But, as thus λ_{min} can be computed with high accuracy from λ_{eff}, a combination of the two cannot give any more information about the absolute magnitude than can λ_{eff} alone. A similar case is presented by the following:—The systematic difference Harvard – Mount Wilson spectral class differs largely for giants and dwarfs (38), and as the correlation constant between H and W is only 0·932, it would be possible to conclude whether a star is a giant or a dwarf if its spectral class in both the Harvard and Mount Wilson systems is known. It is evident that this cannot be so.

That there is, nevertheless, a change in λ_{eff} and λ_{min} with absolute magnitude is clearly shown in figs. 3 and 4, which have been arranged in such a way as to correspond approximately to the usual spectral-class-luminosity diagram. However, it appears from these that λ_{eff} is more suitable for the indication of M_{abs} than is λ_{min}, which fact is quite natural and results from the more accurate measuring of λ_{eff}. It may also be remarked that as shown in fig. 3, λ_{min} apparently does not change appreciably with the absolute magnitude.

From the data represented in fig. 4 Lindblad has suggested that stars

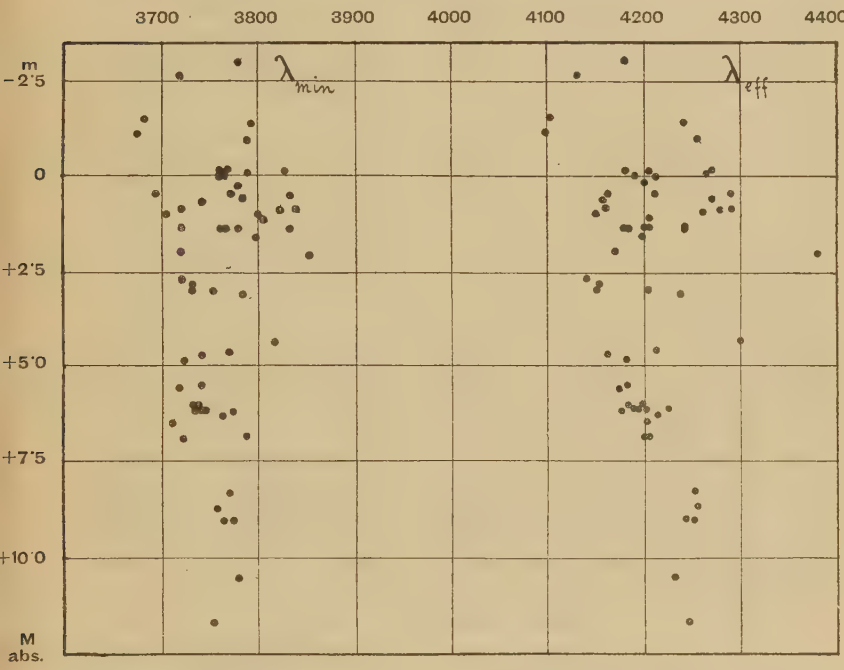

Fig. 3. Fig. 4.

with λ_{eff} larger than 4260 Å on his system are red giants. With the fact in view, however, that for many observers the effective wave-length decreases again from K5 to Mc (giants) we find that the number of stars thus separated from the rest as giants is unfortunately very small and practically restricted to K2–K5. If we also take account of the fact that the personal difference in λ_{eff} for late-type stars can amount to as much as 86 Å (between Bergstrand and Lindblad, same instrument, same kind of plates) we must regard the evidence mentioned here, although extremely interesting, as only preliminary and in need of confirmation.

It is interesting, therefore, to compare with this a statement of Seares (42), who arrives at the conclusion that "the problem of finding the luminosity of a star from observations of colour alone seems to

be indeterminate, owing to the fact that the effect of M_{abs} on colour enters in such a manner that it cannot be separated from that due to spectral class."

On the other hand, it seems not entirely impossible to obtain knowledge of the luminosity of a star if both the spectral class and λ_{eff} are known. It is in this case sufficient to know whether the star is a giant or a dwarf, for Hertzsprung and Russell (43) have shown that the luminosity of a dwarf is so closely related to the spectral class that from a knowledge of the latter quantity we can make a fair estimate of the former. The correlation constant for these two quantities comes out as high as 0·730, thus fully justifying the above-cited conclusion. (For this computation it was necessary to increase the scale in spectral class considerably between K and M, in order to make the relation between M_{abs} and Sp as closely linear as possible.) As giants are extremely rare among the F and early G stars (first pointed out by Hertzsprung), the fainter stars of these classes can generally be regarded as dwarfs. The M dwarfs must, on account of their low luminosity, betray their presence by a large value of the reduced proper motion (the quantity $m + 5 + 5 \log \mu$); the brighter stars of this spectral class can therefore be treated as giants. (The New Henry Draper Catalogue contains probably not more than 20–30 dwarfs in a total number of about 3800 M stars.)

It is therefore a desideratum to develop criteria of luminosity for the stars between spectral classes G5 and K5, and it would be extremely valuable if future development of effective wave-length methods could accomplish something in this direction. Lindblad's criterion, if confirmed, looks very promising, as this seems to work exactly for those spectral classes where it is needed most. But at present it seems to be out of the question to derive any such general criteria without first establishing a standard scale of effective wave-lengths to which all observers could reduce their measures. Such work has already been taken up by Professor Bergstrand, according to private information obtained from him.

Summary.

1. So long as no standard scale of effective wave-lengths has been established, and their determination is subject to large personal equations and systematic differences which have been but meagrely investigated, they cannot be regarded as equivalent to spectral class in individual cases.

2. A thorough and extensive investigation as to all sources of systematic differences affecting measures of effective wave-length is highly desirable.

3. When it becomes possible to master these difficulties, effective wave-lengths may prove valuable as substitutes for spectral classes in dealing with faint stars, and in indicating luminosities when used in connection with known spectral classes.

4. Instead of Lindblad's λ_{min}, a λ_{min} obtained through ultraviolet screens is suggested (for application with reflectors only).

5. Investigation of a possible relation between λ_{min}, λ_{eff}, and the

intensity measures in short spectra (44) will be of importance. In this connection the wave-length of maximum intensity, as determined from shortly exposed low-dispersion slit-spectrograms, may be used as a substitute for λ_{eff}.

Lick Observatory:
1922 *May* 8.

BIBLIOGRAPHY.

(1) Names like Ἀνταρης, cf. Schiaparelli; Rubra Canicula, *Atti dell' R. Acc. di Sc.*, ser. iii., vol. ii., fasc. i., vol. iii., fasc. i., 1896–97; also Fr. Boll, *Abhand. K. Bayr. Ak. Wiss.*, **30**, i., 1916.
(2) For a Discussion of Different Colour-scales, see Hagen, *Ap. J.*, **34**, 261, 1911.
(3) *Lick Obs. Bull.*, **8**, 104; *Bull. Bur. Stand.*, **11**, 613, 1914.
(4) *Publ. A.S.P.*, **27**, 159, 1915.
(5) *Mt. Wilson Report*, 1920.
(6) Guthnick and Hügeler, *Veröff. Berlin-Babelsberg*, Bd. **2**, Th. 3, 30, and *A.N.*, **210**, 345.
(7) *Bull. Astr.*, **26**, 5, 1909; *C.R.*, **149**, 557, 1038, 1909; **173**, 72, 380, 1921.
(8) *Proc. Nat. Acad. Sc.*, **2**, 521, 1916; *Mt. Wilson Comm.*, No. **33**, 1916.
(9) *Pop. Astr.*, **27**, 30, 1919.
(10) *V.J.S.*, **56**, 206, 1921; *A.N.*, **214**, 1, 1921.
(11) *Mt. Wilson Contr.*, No. **97**.
(12) *K. Svenska Vetensk. Ak. Handl.*, Bd. **61**, No. 15, 97, 1921.
(13) *Ap. J.*, **41**, 234, 1915.
(14) *Proc. Amst. Acad.*, **16**, 149, 1913; *V.J.S.*, **56**, 158, 1921, cf. Schilt, *B.A.N.*, **1**, 51, 1922.
(15) v. Zeipel and Lindgren, *loc. cit.*, pp. 7 and 19.
(16) Yerkes Actinometry, *Ap. J.*, **36**, 169, 1912.
(17) Göttingen Aktinometrie, *Astr. Mitt. Sternw. Gött.*, **14**, 42, 1912.
(18) *H.A.*, **59**.
(19) "Studies based on the Colours and Magnitudes in Stellar Clusters," Various, *Mt. Wilson Contributions and Communications.*
(20) *Mt. Wilson Report*, 1921.
(21) *Ap. J.*, **5**, 25, 1897.
(22) *Conférence Astrophotogr. internat de juillet*, 1900, p. 41, 1901.
(23) *A.N.*, **139**, 353, 1896, cf. *A.N.*, **183**, 297, 299, 1909.
(24) *Bull. Astr.*, **25**, 5, 1908, cf. *A.N.*, **186**, 177, 1910.
(25) "Recherches sur les couleurs d'étoiles fixes," *Nova Acta Reg. Soc. Sc.*, Upsala, ser. 4, vol. ii., No. 4, 1909.
(26) *Arkiv for Mat. Astr. och Fysik*, **11**, No. 17, 1916; **13**, No. 26, 1918.
(27) *Ap. J.*, **46**, 205, 1917; **50**, 376, 1919; *A.N.*, **207**, 73, 1918.
(28) *A.N.*, **213**, 49, 1921.
(29) *M.N.*, **82**, 65, 1921.
(30) *A.N.*, **215**, 233, 1922.
(31) *A.N.*, **177**, 241, 1908.
(32) *A.N.*, **213**, 329, 1921.
(33) *Ap. J.*, **53**, 349, 1921.
(34) Smart and Green, *M.N.*, **81**, 179, 1921.
(35) G. H. ten Bruggen Cate, *Determinations and Discussion of the Spectral Classes of* 700 *Stars, mostly near the North Pole.* Dissertation, Groningen, 1920.
(36) *Medd. Lunds Astr. Obs.*, ser. ii., No. 19, 1918.
(37) *Mt. Wilson Contr.*, No. **142**.
(38) *Pop. Astr.*, **29**, No. 2.
(39) *Mt. Wilson Contr.* No. **188**.
(40) *Ap. J.*, **49**, 289, 1919.
(41) *Upsala Universitets Årsskrift*, 1920, also Dissertation, Upsala.
(42) *Mt Wilson Report*, 1920, p. 241.
(43) *A.N.*, **179**, 373, 1908; *Pop. Astr.*, **24**, 275, 1914.
(44) Lindblad, *Ap. J.*, **55**, 85, 1922.

Note on the Actual Determination of Photographic and Photovisual Magnitudes. By W. J. Luyten.

As is stated in the preceding paper, photographic magnitudes can be determined only with photographic instruments and with reference to photographic standards; photovisual magnitudes only by means of isochromatic plates together with yellow screens and with reference to visual or photovisual standard stars. There seems need to emphasise the necessity of this, as it has not always been clearly understood.

Thus *photographic* magnitudes of Nova Cygni * have been determined with the aid of the *visual* magnitudes of comparison stars of spectral class F8. It is clear that the magnitudes obtained in this way cannot be called photographic, as they differ from those by the difference in colour-index between the Nova and the comparison stars, and are also dependent on the colour correction of the telescope used. This explains the large discordance between the magnitudes derived in that way and those obtained by the Greenwich observers † by comparison with photographic standards.

One investigator ‡ has determined *photovisual* magnitudes of Nova Cygni by using isochromatic plates *without* yellow filter, and by comparing these with the photographic magnitudes reaches the following statement: "This second curve, which we call photovisual, follows the photographic curve rather closely; however, not much importance should be attached to this." Magnitudes determined in this way do not in the least approach photovisual magnitudes, and the statement mentioned above cannot therefore be considered as remarkable. For it is a well-known fact that the use of an isochromatic plate without yellow filter instead of an ordinary photographic plate will alter the magnitude, *ceteris paribus*, by only a small fraction of the colour-index (probably near one-fiftieth). In fact, this is the basis of Seares's method of exposure ratios.

The Planes of the Spiral Nebulæ in Relation to the Line of Sight. By J. H. Reynolds.

In a paper on "The Galactic Distribution of the Large Spiral Nebulæ," § attention was drawn to the planes of the spiral nebulæ of 10′ in diameter and upwards in relation to the line of sight, and it was shown that there was a considerable excess of these nebulæ inclined at angles under 30°, which angle would be expected to divide the total number equally.

The random distribution of the planes of spirals inclined at all angles over the whole sky is given by the formula

$$n = \mathrm{N}(\sin\theta_1 - \sin\theta_2),$$

where n is the number between any two angles θ_1 and θ_2, and N is the total number considered.

* Smart and Green, *M.N.*, **81**, 179, 1921. † *M.N.*, **81**, 37, 61, 1920.
‡ F. Henroteau, *Journ. R.A.S. Canada*, 1921, p. 155.
§ *M.N.*, **81**, 129.

This formula gives the following number per thousand for angles—

$< 10°$	173
$10°$ and $< 20°$	169
$20°$ and $< 30°$	158
$30°$ and $< 45°$	207
$45°$ and $< 60°$	159
$60°$ to $90°$	134.

Recently a considerable amount of work has been done by Mr. F. G. Brown, who has collected particulars of spirals under 10′ in diameter, and has worked out the angles with the line of sight by comparing the major and minor axes of the ellipses. Owing to the structure of the spiral nebulæ it is impossible to treat these objects rigidly as if they are circular discs. The oblate nucleus in the case of edge-on spirals is often as much as 1′ in diameter, and thus an object appearing in the published lists as $4' \times 1'$ may hardly be inclined at all with the line of sight, whereas the mathematical ratio would give 15°. It is obvious, therefore, that it is impossible to work within narrow limits without introducing large anomalies, especially in the case of spirals inclined at a small angle with the line of sight, and the division of angles already given has been adopted by the writer as the most suitable in view of the degree of accuracy obtainable.

Mr. Brown's examples have been obtained from the *L.O.*, **13**, the *Helwân Bulletins*, 9, 15, 21, and 22, and (at the suggestion of the writer) the list of spirals 3′ and upwards in diameter derived from the Franklin-Adams plates in the Zone $-40°$ to $-90°$.* The material from the Lick and Helwân photographs is comparable from the instrumental point of view, but the small scale of the Franklin-Adams plates does not entitle the southern circumpolar objects to equal weight. At the same time their inclusion makes the total number fairly representative of the whole sky. Mr. Brown's list comprises 263 objects of the following dimensions:—

10′ and over	27
5′ and $<$ 10′	82
3′ and $<$ 5′	154.

The table given below classifies the planes of these objects in relation to the line of sight, the sine of the angle of inclination being obtained by dividing the minor by the major axes of the apparent ellipses.

	0–10°.	10°–20°.	20°<30°.	30°.	>30°–45°.	45°–60°.	60–90°.
10′ and over in diameter	10	6	2	2	3	3	1
Random distribution .	5	5	4	0	6	4	3
5′ to < 10′ in diameter	18	28	7	10	10	2	7
Random distribution .	14	14	13	0	17	13	11
3′ to < 5′ in diameter .	22	45	13	29	27	10	8
Random distribution .	27	26	24	0	31	26	20
Totals . .	50	79	22	41	40	15	16
Random distribution .	46	45	41	0	54	43	34

* *M.N.*, **81**, 598.

It will be noted that there is a marked tendency for the planes of the smaller nebulæ to group between the angles 10°-20°, those 10′ and over being at a maximum between 0°-10°.

The large number appearing as inclined exactly at 30° to the line of sight is almost entirely due to cases where the major axis is approximately twice the minor axis of the ellipse, the dimensions being given in round numbers, such as $6' \times 3'$, $4' \times 2'$, etc. It is, of course, in many instances difficult to give nearer dimensions than this owing to the irregular formation of the spirals, and the indefinite fading of the outer portions, and it would be entirely too rigid an interpretation to infer that they are inclined exactly at 30°. For this reason it is advisable for statistical purposes to divide them equally into two groups, half slightly over and half slightly under 30°. We can then compare the numbers under and over 30° with the number calculated on a random distribution.

	No. under 30°.	Calculated.	No. over 30°.	Calculated.
10′ and over . . .	19	13	8	13
5′ to < 10′ . . .	58	41	24	41
3′ to < 5′ . . .	95	77	59	77
Totals . . .	172	131	91	131

A further comparison can be made of the proportions between the numbers under and over 30°.

10′ and over	2·37 : 1
5′ to < 10′	2·41 : 1
3′ to < 5′	1·61 : 1.

The proportions of the first two groups are very close, and the reduced proportion of the third group is founded on a large number of objects and so seems to be a real effect.

Mr. Brown draws attention to an important paper by Dr. Fath in the *Astronomical Journal*, 28, which deals with small nebulæ photographed at Mount Wilson with the 60-inch reflector.

These nebulæ are found in the "Selected Areas" of Kapteyn and the exposed area of each plate is 1·88 square degrees, included in the regions from +90° to −15°. Altogether 1015 objects are listed with their positions and dimensions. The smallest are 3″ in diameter, and they range up to over 4′, the average being about 16″. Many of these objects are evidently "globular" nebulæ and not spirals, although no doubt of the spiral family. This seems to be proved by the large proportion of examples where the two dimensions given are of the same amount. If these were spirals, the planes inclined at angles between 70° and 90° according to the random distribution formula should be 61 per 1000, whereas over 300 of equal dimensions are found in the lists of 1015 objects, without reckoning those described as stellar for which no dimensions are given, and which are presumably concentric in appearance. These are usually grouped together, and appear on the same plates as those containing elongated nebulæ. The latter are in all

probability spirals, although the spiral structure is rarely to be made out on the plates owing to the small dimensions of the ellipses.

Mr. Brown points out that if we take the 98 objects exceeding 30″ in diameter in the list, there is a large preponderance inclined at less than 30°. If, however, we take the groups—over 120″, 60″ to 120″, 40″ to 60″ and 30″ to 40″—we find a great variation in the proportion over and under 30°. In the first group of 8 the proportions are equal. In the second group of 20, only 4 are over 30°; in the third group of 45, 8 are 30° and over; in the last group of 26 only 2 are over 30°. Yet if we extend the investigation to the objects between 20″ and 30″ we get the result that 41 are under 30°, 6 are of dimensions corresponding exactly to 30°, and 55 are over 30°. This sudden reversal between 20″ and 30″ shows the difficulty of accepting the comparatively few objects given in the paper as evidence of the distribution of the planes at various angles over the whole sky, and we must wait for further investigation of these small nebulæ before they can be included. The total number of small nebulæ of this class is certainly not less than 200,000; indeed, Keeler's estimate was several times this amount.

Conclusion.—It seems evident from the foregoing that the excess of small inclinations of the planes to the line of sight which had already been noted in the case of the larger spirals is continued down to those of 3′ diameter, although in a lessened degree. The average angles of inclination for the three classes are

10′ and upwards	21·9°
5′ to < 10′	24·2°
3′ to < 5′	26·7°.

As such a systematic arrangement of the planes indicates a relationship between their distances and our position in the Galaxy, the evidence is definitely against the spiral nebulæ being comparable in size with our galactic system, and in favour of the views advocated by Shapley, in which the Galaxy is regarded as of far greater dimensions than the spirals.

The Theory of Astronomical Interferometer Measurements.

By H. Spencer Jones, M.A., B.Sc.

1. The theory of the application of the interferometer to astronomical measurements was first worked out for some special cases by Michelson (*Phil. Mag.*, 30, 1, 1890), with the limitation that the width of the slits was negligible compared with their distance apart. In practice it is not possible in general to adhere to this limitation, as under such circumstances sufficient light is not transmitted for the interference fringes to be visible. Hamy (*Bull. Astron.*, 16, 257, 1899) discussed the theory of the measurement, by interferometer methods, of the small angular diameter of a uniformly illuminated circular object, in the more general case when the width of the slits cannot be neglected in com-

parison with their separation, and found that an appreciable correction to the formula found by Michelson was necessitated. In the present paper the discussion of a number of different cases is given, carrying the theory to a more complex stage than heretofore.

2. *General Expression for the Illumination in the Focal Plane.*—We assume that the two apertures are rectangular, equal, and parallel. Their length, width, and separation are denoted respectively by h, a, l.

Rectangular axes Ox, Oy, Oz are taken such that the origin O is equidistant from the two slits, and the axes Oy, Oz are in the plane of the slits, Oz being parallel and Oy perpendicular to their length. The axis Oy passes through the mid-points of their long sides. The direction in which a luminous surface situated at infinity is seen is defined by the angles ω, ψ, ω being measured in the plane Oxy from Ox towards Oy and ψ being measured from this plane towards Oz. The waves from this point passing through the two apertures produce a disturbance at infinity in the direction θ, ϕ, the angles θ, ϕ being measured similarly to ω, ψ respectively.

The angles θ, ϕ, ω, ψ are so small in practice that their cosines may be placed equal to unity and their sines equal to the circular measure of the angles. Under these circumstances it follows at once, from the theory of diffraction, that the intensity of illumination in the direction θ, ϕ produced by a small area at ω, ψ may be taken as

$$J^2 \cos^2 \frac{\pi l}{\lambda}(\omega-\theta)\left[\frac{\sin\frac{\pi a}{\lambda}(\omega-\theta)}{\frac{\pi a}{\lambda}(\omega-\theta)}\right]^2\left[\frac{\sin\frac{\pi h}{\lambda}(\phi-\psi)}{\frac{\pi h}{\lambda}(\phi-\psi)}\right] d\omega\, d\psi \quad . \quad (1)$$

where J^2 is proportional to the intrinsic luminosity of the source (see Hamy, *loc. cit.*).

The illumination produced by an extended surface at the point θ, ϕ is obtained by integrating this expression with regard to ω, ψ over the surface in question. The factor $\sin^2\frac{\pi h}{\lambda}(\phi-\psi)\Big/\left[\frac{\pi h}{\lambda}(\phi-\psi)\right]^2$ expresses the variation in intensity in the direction parallel to the slits. Its maximum value is unity. For the majority of the applications of the theory which we require to discuss, it is necessary to consider only the most luminous portion of the field, and in such cases we may replace this factor by unity.

The general expression for the intensity then becomes

$$I = \int d\omega \int J^2 \left[\frac{\sin\frac{\pi a}{\lambda}(\omega-\theta)}{\frac{\pi a}{\lambda}(\omega-\theta)}\right]^2 \cos^2\frac{\pi l}{\lambda}(\omega-\theta) d\psi \quad . \quad . \quad (2)$$

J being a function of ψ and ω.

3. *A Spherical, uniformly Illuminated Star.*—We consider first the case in which the object appears as a circular disc with uniform intrinsic illumination. If its angular diameter is denoted by ϵ and its centre is

on the axis Ox in the negative direction, its apparent boundary is given by the equation

$$\psi^2 + \omega^2 = \frac{\epsilon^2}{4}.$$

In this case formula (2) gives, since J is constant and may be placed equal to unity,

$$I = \int_{-\frac{\epsilon}{2}}^{+\frac{\epsilon}{2}} 2\psi \left[\frac{\sin \frac{\pi a}{\lambda}(\omega - \theta)}{\frac{\pi a}{\lambda}(\omega - \theta)} \right]^2 \cos^2 \frac{\pi l}{\lambda}(\omega - \theta) d\omega \quad . \quad . \quad (3)$$

We write

$$\omega = \frac{\epsilon}{2} . u \qquad \frac{\pi l \epsilon}{2\lambda} = m \qquad \frac{a}{l} = p.$$

Then

$$I = \frac{\epsilon^2}{2} \int_{-1}^{+1} \sqrt{1 - u^2} \left[\frac{\sin p\left(mu - \frac{\pi l \theta}{\lambda}\right)}{p\left(mu - \frac{\pi l \theta}{\lambda}\right)} \right]^2 \cos^2 \left(mu - \frac{\pi l \theta}{\lambda}\right) du.$$

$p = a/l$ is a measure of the ratio of the width of the slits to their distance apart. We assume it to be finite, but sufficiently small for powers above the second to be neglected. With this assumption

$$I = \frac{\epsilon^2}{2} \int_{-1}^{+1} \sqrt{1 - u^2} \left\{ 1 - \tfrac{1}{3} p^2 \left(mu - \frac{\pi l \theta}{\lambda}\right)^2 \right\} \left\{ 1 + \cos 2\left(mu - \frac{\pi l \theta}{\lambda}\right) \right\} du$$

$$= \epsilon^2 \int_0^1 \sqrt{1 - u^2} \left\{ 1 - \tfrac{1}{3} p^2 \left(m^2 u^2 + \frac{\pi^2 l^2 \theta^2}{\lambda^2} \right) \right\} \left(1 + \cos 2mu \cos \frac{2\pi l \theta}{\lambda} \right) du$$

$$+ \epsilon^2 \int_0^1 \sqrt{1 - u^2} . \tfrac{2}{3} mp . \frac{\pi l \theta}{\lambda} . \sin 2mu \sin \frac{2\pi l \theta}{\lambda} du.$$

We write

$$\int_0^1 \sqrt{1 - u^2} \cos 2mu \, du = \frac{\pi}{4} A_0$$

$$\int_0^1 \sqrt{1 - u^2} . u^2 . \cos 2mu \, du = \frac{\pi}{4} A_1$$

$$\int_0^1 \sqrt{1 - u^2} . u . \sin 2mu \, du = \frac{\pi}{4} B_1.$$

Then we have

$$I = \frac{\pi \epsilon^2}{4} \left\{ 1 - \frac{p^2 m^2}{12} - \frac{p^2}{3} . \frac{\pi^2 l^2 \theta^2}{\lambda^2} \right\} + \frac{\pi \epsilon^2}{4} A_0 \left(1 - \tfrac{1}{3} \frac{p^2 \pi^2 l^2 \theta^2}{\lambda^2} \right) \cos \frac{2\pi l \theta}{\lambda}$$

$$- \frac{\pi \epsilon^2}{4} . A_1 . \tfrac{1}{3} m^2 p^2 . \cos \frac{2\pi l \theta}{\lambda} + \frac{\pi \epsilon^2}{4} B_1 \tfrac{2}{3} mp . \frac{\pi l \theta}{\lambda} \sin \frac{2\pi l \theta}{\lambda} \quad (4)$$

The constant factor $\frac{\pi \epsilon^2}{4}$ in this expression can be neglected.

In the special case of infinitely narrow slits ($p = 0$) we have

$$I = 1 + A_0 \cos \frac{2\pi l\theta}{\lambda}$$

$$= 1 + \frac{4}{\pi}\int_0^1 \sqrt{1-u^2} \cos 2mu\, du \,.\, \cos \frac{2\pi l\theta}{\lambda}.$$

For a given value of $m(\pi l\epsilon/2\lambda)$, this expression has its maxima and minima values at the positions given by

$$\theta = \frac{n}{2}\frac{\lambda}{l},$$

where n is an integer. Whether the even values of θ correspond to the maxima or minima of I depends upon the value of m.

The integral A_0 is known to be zero for the special value of $m = 1{\cdot}22\frac{\pi}{2}$. In this case the illumination near the centre of the field becomes uniform, and the system of interference fringes which correspond to the maxima and minima of I therefore disappear. For this value of m we have

$$m = \frac{\pi l\epsilon}{2\lambda} = 1{\cdot}22\frac{\pi}{2}$$

or

$$\epsilon = 1{\cdot}22\frac{\lambda}{l}.$$

In other words, if the interference fringes vanish for a distance of the slits, l, the angular diameter of the object under observation is $\epsilon = 1{\cdot}22\lambda/l$. This is the result obtained by Michelson.

If, however, p cannot be neglected, this result no longer holds, the variation in intensity produced by the factor $\sin^2 \frac{\pi a}{\lambda}(\omega-\theta)\Big/\left[\frac{\pi a}{\lambda}(\omega-\theta^2\right]$ in (3) coming into play. We require to determine the value of m, which in this case produces uniform illumination near the centre. We do this by expressing the condition that the central maximum and minimum intensities shall become equal, and solving the resultant equation for m.

To the order of approximation which we have adopted we may write

$$I = \text{const.} + f(\theta) + p^2\phi(\theta).$$

When $p = 0$, suppose that $\theta = \xi$ corresponds to a maximum or minimum value of I. Then

$$f'(\xi) = 0.$$

When $p \neq 0$, suppose that the corresponding maximum or minimum is given by $\theta = \xi + kp^2$. k is determined by

$$f'(\xi + kp^2) + p^2\phi(\xi + kp^2) = 0$$

or

$$k = -\phi'(\xi)/f''(\xi).$$

The value of I corresponding to this value of θ is

$$\begin{aligned} \mathrm{I} &= \text{const.} + f(\xi + kp^2) + p^2\phi(\xi + kp^2) \\ &= \text{const.} + f(\xi) + p^2\phi(\xi) \end{aligned}$$

to the order of approximation p^2.

Hence if $\theta = \xi$ corresponds to a stationary value of I when $p = 0$, the value of I for the corresponding stationary value when $p \neq 0$ is $\mathrm{I}(\xi)$, obtained by using the same value of θ.

In the problem under consideration the central maxima and minima when $p = 0$ correspond to the values of θ given by $\theta = 0$ and $\frac{\lambda}{2l}$. The condition for uniform intensity at the centre of the field when p is not zero is therefore

$$\mathrm{I}(0) = \mathrm{I}\left(\frac{\lambda}{2l}\right),$$

or, from (4),

$$\mathrm{A}_0 - \tfrac{1}{3}m^2p^2\mathrm{A}_1 = -\frac{p^2\pi^2}{12} - \mathrm{A}_0\left(1 - \frac{p^2\pi^2}{12}\right) + \tfrac{1}{3}m^2p^2\mathrm{A}_1,$$

or

$$24\mathrm{A}_0 - p^2[-\pi^2 + \pi^2\mathrm{A}_0 + 8m^2\mathrm{A}_1] = 0.$$

Now

$$\begin{aligned} \frac{\pi}{4}(\mathrm{A}_0 - \mathrm{A}_1) &= \int_0^1 (1 - u^2)^{\frac{1}{2}} \cos 2mu\, du \\ &= \frac{3}{2m}\int_0^1 (1 - u^2)^{\frac{1}{2}} u \sin 2mu\, du, \text{ integrating by parts} \\ &= \frac{3}{2m} \cdot \frac{\pi}{4}\mathrm{B}_1, \end{aligned}$$

or

$$\begin{aligned} \mathrm{A}_1 &= \mathrm{A}_0 - \frac{3}{2m}\mathrm{B}_1 \\ &= \mathrm{A}_0 + \frac{3}{4m}\frac{d\mathrm{A}_0}{dm}, \end{aligned}$$

so that

$$24\mathrm{A}_0 - p^2\left[-\pi^2 + (\pi^2 + 8m^2)\mathrm{A}_0 + 6m\frac{d\mathrm{A}_0}{dm}\right] = 0.$$

When $p = 0$, $m = 1{\cdot}22 \times \frac{\pi}{2}$. Assume $m = 1{\cdot}22 \times \frac{\pi}{2} + kp^2$.

Then, since $\mathrm{A}_0\left(1{\cdot}22\frac{\pi}{2}\right) = 0$

$$24kp^2\mathrm{A}_0'\left(1{\cdot}22 \times \frac{\pi}{2}\right) = p^2\left[-\pi^2 + 3{\cdot}66\pi\mathrm{A}_0'\left(1{\cdot}22\frac{\pi}{2}\right)\right].$$

Hamy gives $-{\cdot}4176$ as the value for $\mathrm{A}_0'\left(1{\cdot}22\frac{\pi}{2}\right)$, from which it follows that $k = 1{\cdot}47$.

Thus

$$\epsilon = 1{\cdot}22\,\frac{\lambda}{l}\left\{1 + 0{\cdot}765\left(\frac{a}{l}\right)^2\right\}$$

is the formula connecting the angular diameter with the distance apart of the slits for which the central fringe vanishes, when the width of the slits cannot be neglected. This result is in agreement with that found by Hamy.

4. *The Case of an Ellipsoidal Star.*—If the star is not spherical but ellipsoidal, its apparent contour will be an ellipse whose equation may be represented by

$$a\psi^2 + 2b\psi\omega + c\omega^2 = 1.$$

Formula (2) gives in this case since, for a given value of ω, the difference of the two limiting values of ψ is

$$\{a - (ac - b^2)\omega^2\}^{\frac{1}{2}}/a$$

$$\mathrm{I} = \int_{-\frac{\epsilon}{2}}^{+\frac{\epsilon}{2}} \frac{2}{a}\{a - (ac - b^2)\omega^2\}^{\frac{1}{2}} \cos^2\frac{\pi l}{\lambda}(\omega - \theta)d\omega.$$

We have put $p = 0$; *i.e.* the slits are assumed to be very narrow. In the right-hand side ϵ denotes the angular diameter measured at right angles to the slits, *i.e.* the angular distance between the tangents to the contour parallel to the slits.

Put

$$(ac - b^2)\omega^2 = au^2$$

or

$$\omega = \frac{\epsilon}{2}u,$$

since $[a/ac - b^2]^{\frac{1}{2}}$ is the semidiameter perpendicular to the slits. Then

$$\mathrm{I} = \frac{\epsilon}{a^{\frac{1}{2}}}\int_{-1}^{+1}\sqrt{1 - u^2}\left\{1 + \cos\left(\frac{\pi l\epsilon}{\lambda}u - \frac{2\pi l}{\lambda}\theta\right)\right\}du$$

$$= \frac{2\epsilon}{a^{\frac{1}{2}}}\int_0^1\sqrt{1 - u^2}\left\{1 + \cos\frac{\pi l\epsilon}{\lambda}u\,.\cos\frac{2\pi l}{\lambda}\theta\right\}du;$$

or, ignoring the constants,

$$\mathrm{I} = 1 + \mathrm{A}_0\cos\frac{2\pi l\theta}{\lambda},$$

exactly as in the case of the circular disc.

The condition for the vanishing of the fringes is, therefore, as before

$$\epsilon = 1{\cdot}22\,\frac{\lambda}{l},$$

where now ϵ denotes the angular diameter measured in a direction perpendicular to the slits.

The discussion of the correction required by this formula to allow for a finite width of slit is identical with that for the circular disc, so

that the formula obtained in the preceding section will apply also to this case.

By rotating the slits and determining ϵ in various orientations, it should therefore be possible to test whether the star or other body under examination has an appreciable ellipticity.

5. *Spherical Star, with Darkening towards the Limb.*—The preceding results were derived on the assumption that the apparent intrinsic luminosity was uniform over the disc. In general this will not be the case. The corresponding results for any particular law of darkening can only be derived by the method of quadratures, though it is possible to see without difficulty what will be the general effect of a falling off in surface luminosity from centre of disc to limb.

We place $J^2=f\left(\frac{\pi}{\epsilon}\sqrt{\psi^2+\omega^2}\right)$, f being a continuous function which decreases gradually in magnitude as its argument increases from 0 to $\frac{\pi}{2}$. $\sqrt{\psi^2+\omega^2}$ represents the apparent angular distance of any point from the centre of the disc. From symmetry considerations it is evident that $f(x)=f(-x)$. At the centre $J^2=f(0)$; at the limb $J^2=f\left(\frac{\pi}{2}\right)$.

The component of the intensity which depends upon θ is

$$I_1=\int d\omega\int f\left(\frac{\pi}{\epsilon}\sqrt{\psi^2+\omega^2}\right)\cos\frac{2\pi l\omega}{\lambda}\cos\frac{2\pi l\theta}{\lambda}\,d\psi.$$

Since

$$f\left(\frac{\pi}{\epsilon}\omega\right)>f\left(\frac{\pi}{\epsilon}\sqrt{\psi^2+\omega^2}\right)>f\left(\frac{\pi}{2}\right),$$

we have

$$\frac{\epsilon^2}{2}\int_0^1 f\left(\frac{\pi u}{2}\right)\sqrt{1-u^2}\cos\left(\frac{\pi l\epsilon}{\lambda}u\right)du\,.\,\cos\frac{2\pi l\theta}{\lambda}$$

$$>I_1>\frac{\epsilon^2}{2}\int_0^1 f\left(\frac{\pi}{2}\right)\cdot\sqrt{1-u^2}\cos\left(\frac{\pi l\epsilon}{\lambda}u\right)du\,.\,\cos\frac{2\pi l\theta}{\lambda}.$$

I_1 will vanish for a value of $l\epsilon$ intermediate between the values which cause the right- and left-hand expressions to vanish:

$$\int_0^1\sqrt{1-u^2}\cos\frac{\pi l\epsilon}{\lambda}u\,.\,du \text{ vanishes if } \frac{\pi l\epsilon}{\lambda}=1{\cdot}22\,.\,\pi.$$

The integrand is then positive for a range in the values of u from 0 to $\frac{1}{1{\cdot}22}$, and negative for the range $\frac{1}{1{\cdot}22}$ to 1.

In the left-hand integral the integrand is equal to that in the right-hand integral multiplied by a factor whose value is greater in the former

range than in the latter. Hence for the vanishing of this integral the integrand must change sign for a smaller value of u than $\frac{1}{1\cdot22}$. In other words, it vanishes if $\frac{\pi l\epsilon}{\lambda} > 1\cdot22\,\pi$. Hence also I_1 must vanish, or the interference fringes will disappear for a value of $l\epsilon$ such that

$$\epsilon > 1\cdot22\,\frac{\lambda}{l}.$$

Michelson (*loc. cit.*) discussed the case in which the darkening towards the limb followed the same law as in the case of the sun, and found, by quadratures, that when the interference fringes vanished

$$\epsilon = 1\cdot33\,\frac{\lambda}{l}.$$

This particular case shows that the correction necessitated by the darkening may be appreciable.

6. *Double Star, consisting of two Equal Stars of the same Intrinsic Brightness.*—We consider next the case in which the object is a double star, the two components being circular, of the same angular diameter and surface brightness. The common angular diameter is denoted by ϵ and the angular distance apart of the centres by α. The origin is taken on the line of and midway between the centres, the line being taken as the axis Oy, perpendicular to the slits.

The boundaries of the two stars are then

$$\psi^2 + \left(\omega - \frac{\alpha}{2}\right)^2 = \frac{\epsilon^2}{4}$$

$$\psi^2 + \left(\omega - \frac{\alpha}{2}\right)^2 = \frac{\epsilon^2}{4}.$$

Writing, as before, $\frac{\pi l\epsilon}{2\lambda} = m$, $\frac{\alpha}{l} = p$, equation (3) gives

$$I = \int_{\frac{\alpha}{2}-\frac{\epsilon}{2}}^{\frac{\alpha}{2}+\frac{\epsilon}{2}} 2\psi \left[\frac{\sin\frac{\pi a}{\lambda}(\omega-\theta)}{\frac{\pi a}{\lambda}(\omega-\theta)}\right]^2 \cos^2\frac{\pi l}{\lambda}(\omega-\theta)\,d\omega$$

$$+ \int_{-\frac{\alpha}{2}-\frac{\epsilon}{2}}^{-\frac{\alpha}{2}+\frac{\epsilon}{2}} 2\psi \left[\frac{\sin\frac{\pi a}{\lambda}(\omega-\theta)}{\frac{\pi a}{\lambda}(\omega-\theta)}\right]^2 \cos^2\frac{\pi l}{\lambda}(\omega-\theta)\,d\omega,$$

the two integrals applying to the two separate stars.

Writing in the first $\left(\omega-\frac{\alpha}{2}\right)=\frac{\epsilon}{2}u$, and in the second $\left(\omega+\frac{\alpha}{2}\right)=\frac{\epsilon}{2}u$:

$$I=\frac{\epsilon^2}{2}\int_{-1}^{+1}(1-u^2)^{\frac{1}{2}}\left[\frac{\sin p\left(mu-\frac{\pi l\theta}{\lambda}-\frac{\pi l\alpha}{2\lambda}\right)}{p\left(mu-\frac{\pi l\theta}{\lambda}-\frac{\pi l\alpha}{2\lambda}\right)}\right]^2\cos^2\left(mu-\frac{\pi l\theta}{\lambda}-\frac{\pi l\alpha}{2\lambda}\right)du$$

$$+\frac{\epsilon^2}{2}\int_{-1}^{+1}(1-u^2)^{\frac{1}{2}}\left[\frac{\sin p\left(mu-\frac{\pi l\theta}{\lambda}+\frac{\pi l\alpha}{2\lambda}\right)}{p\left(mu-\frac{\pi l\theta}{\lambda}+\frac{\pi l\alpha}{2\lambda}\right)}\right]^2\cos^2\left(mu-\frac{\pi l\theta}{\lambda}+\frac{\pi l\alpha}{2\lambda}\right)du \quad (5)$$

Expanding as before to the second power of p and reducing, neglecting the numerical factor $\frac{\epsilon^2}{2}$,

$$I=\int_0^1(1-u^2)^{\frac{1}{2}}\left[1-\tfrac{1}{3}p^2\left(m^2u^2+\frac{\pi^2l^2\theta^2}{\lambda^2}+\frac{\pi^2l^2\alpha^2}{4\lambda^2}\right)\right]du$$

$$+\int_0^1(1-u^2)^{\frac{1}{2}}\left[1-\tfrac{1}{3}p^2\left(m^2u^2+\frac{\pi^2l^2\theta^2}{\lambda^2}+\frac{\pi^2l^2\alpha^2}{4\lambda^2}\right)\right]\cos\frac{2\pi l\theta}{\lambda}\cos\frac{\pi l\alpha}{\lambda}\cos 2mu\,du$$

$$+\int_0^1(1-u^2)^{\frac{1}{2}}\tfrac{2}{3}p^2\cdot\frac{\pi^2l^2}{2\lambda^2}\theta\,.\,\alpha\,.\sin\frac{2\pi l\theta}{\lambda}\sin\frac{\pi l\alpha}{\lambda}\cos 2mu\,du$$

$$+\int_0^1(1-u^2)^{\frac{1}{2}}\,.\,\tfrac{2}{3}mp^2\,.\,\frac{\pi l\theta}{\lambda}\,.\sin\frac{2\pi l\theta}{\lambda}\,.\cos\frac{\pi l\alpha}{\lambda}\sin 2mu\,.\,udu$$

$$+\int_0^1(1-u^2)^{\frac{1}{2}}\,.\,\tfrac{2}{3}mp^2\,.\,\frac{\pi l\alpha}{2\lambda}\,.\cos\frac{2\pi l\theta}{\lambda}\,.\sin\frac{\pi l\alpha}{\lambda}\sin 2mu\,.\,udu,$$

and A_0, A_1, B_1 having the same significance as previously,

$$I=\frac{\pi}{4}\left\{1-\tfrac{1}{3}p^2\cdot\frac{\pi^2l^2}{\lambda^2}\left(\theta^2+\frac{\alpha^2}{4}\right)\right\}-\tfrac{1}{3}m^2p^2\cdot\frac{\pi}{16}$$

$$+\frac{\pi}{4}A_0\left\{1-\tfrac{1}{3}p^2\frac{\pi^2l^2}{\lambda^2}\left(\theta^2+\frac{\alpha^2}{4}\right)\right\}\cos\frac{2\pi l\theta}{\lambda}\cos\frac{\pi l\alpha}{\lambda}+\frac{\pi}{4}A_0\cdot\tfrac{2}{3}p^2\cdot\frac{\pi^2l^2}{\lambda^2}\cdot\frac{\theta\alpha}{2}\sin\frac{2\pi l\theta}{\lambda}\sin\frac{\pi l\alpha}{\lambda}$$

$$-\frac{\pi}{4}\cdot A_1\cdot\tfrac{1}{3}m^2p^2\cdot\cos\frac{2\pi l\theta}{\lambda}\cos\frac{\pi l\alpha}{\lambda}+\frac{\pi}{4}B_1\cdot\tfrac{2}{3}mp^2\cdot\frac{\pi l\theta}{\lambda}\sin\frac{2\pi l\theta}{\lambda}\cdot\cos\frac{\pi l\alpha}{\lambda}$$

$$+\frac{\pi}{4}B_1\cdot\tfrac{2}{3}mp^2\cdot\frac{\pi l\alpha}{2\lambda}\cos\frac{2\pi l\theta}{\lambda}\sin\frac{\pi l\alpha}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If $p=0$,

$$\frac{4}{\pi}I=1+A_0\cos\frac{\pi l\alpha}{\lambda}\cdot\cos\frac{2\pi l\theta}{\lambda}\,.$$

For a given value of l the term depending on θ vanishes if $\theta = \frac{2n+1}{2} \cdot \frac{\lambda}{2l}$ and has maxima and minima at the values of θ given by $\theta = \frac{n}{2}\frac{\lambda}{l}$. There is thus, in general, a series of equidistant interference fringes at the centre of the field.

The term $\cos\frac{\pi l\alpha}{\lambda}$ vanishes if $\alpha = \frac{\lambda}{2l}$. The function A_0 vanishes if $\frac{l\epsilon}{\lambda} = 1{\cdot}22$. If $\alpha = \frac{\lambda}{2l}$ the value of $\frac{l\epsilon}{\lambda}$ is $\frac{\epsilon}{2\alpha}$, and since ϵ is relatively small compared with α, the value of $\epsilon/2\alpha$ is much smaller than $1{\cdot}22$. Therefore, if l is gradually increased from zero, $\cos\pi l\alpha/\lambda$ will vanish before A_0. For the value of l given by $\alpha = \frac{\lambda}{2l}$ the intensity near the centre of the field becomes uniform, and the interference fringes disappear. In this way the angular separation of the double star is given by

$$\alpha = \frac{\lambda}{2l}.$$

In the general case when a/l is not zero, $\theta = 0$ still makes I a maximum or minimum, since $\frac{dI}{d\theta_{\theta=0}}$ vanishes. The succeeding minimum or maximum will occur at a value of $\theta = \frac{\lambda}{2l} + kp^2$, and corresponds to the root of $\frac{dI}{d\theta}$ next after zero. As in the case of the single star, the value of $I(\theta)$ corresponding to this value of θ is $I\left(\frac{\lambda}{2l}\right)$. The condition for the central maximum and minimum to be equal is therefore

$$I(0) = I\left(\frac{\lambda}{2l}\right),$$

and from this equation is to be derived the value of α which corresponds to the value $\alpha = \lambda/2l$ when $p = 0$.

The condition $I(0) = I\left(\frac{\lambda}{2l}\right)$ gives, on substitution in (6) and reduction,

$$\frac{\pi^2p^2}{12} + A_0\cos\frac{\pi l\alpha}{\lambda}\left(2 - \tfrac{2}{3}p^2\cdot\frac{\pi^2l^2\alpha^2}{4\lambda^2} - \frac{p^2\pi^2}{12}\right) - \tfrac{2}{3}A_1\cdot m^2p^2\cos\frac{\pi l\alpha}{\lambda} + \tfrac{4}{3}B_1mp^2\cdot\frac{\pi l\alpha}{2\lambda}\sin\frac{\pi l\alpha}{\lambda} = 0.$$

If $\alpha = 0$, this condition reduces, as it should, to the equation previously obtained for a single star.

To solve this equation we assume

$$\frac{\pi l\alpha}{\lambda} = \frac{\pi}{2} + kp^2.$$

Then, neglecting powers of p higher than the second,

$$\frac{\pi^2 p^2}{12} - 2A_0 . kp^2 + \tfrac{4}{3} . B_1 . mp^2 . \frac{\pi}{4} = 0$$

or

$$k = \frac{\pi^2}{24 A_0} + \frac{\pi}{6} . m \frac{B_1}{A_0} .$$

The value of m is $\frac{\pi l \epsilon}{2\lambda}$ or $\frac{\pi \epsilon}{4a}$, and is therefore small unless the two stars are at a distance apart which does not greatly exceed their diameters. For small values of m we may put

$$A_0 = \frac{4}{\pi}\int_0^1 (1-u^2)^{\frac{1}{2}} \cos 2mu\, du = 1 - \frac{m^2}{2}$$

$$B_1 = \frac{4}{\pi}\int_0^1 (1-u^2)^{\frac{1}{2}} u \sin 2mu\, du = \frac{m}{2} .$$

Then

$$k = \frac{\pi^2}{24}\left(1 + \frac{m^2}{2}\right) + \frac{\pi}{6} . \frac{m^2}{2} .$$

Neglecting $\left(\frac{\epsilon}{a}\right)^2$, we have

$$\frac{\pi l a}{\lambda} = \frac{\pi}{2} + \frac{\pi^2}{24}\left(\frac{a}{l}\right)^2$$

or

$$a = \frac{\lambda}{2l}\left\{1 + \frac{\pi}{12}\left(\frac{a}{l}\right)^2\right\} = \frac{\lambda}{2l}\left\{1 + \cdot 262\left(\frac{a}{l}\right)^2\right\}.$$

The correction factor for finite width of aperture is therefore in this case considerably smaller than in the case of the single star.

The result shows also how the simple formula $a = \frac{\lambda}{2l}$ requires to be modified if the diameters of the component stars cannot be neglected in comparison with their distance apart. If ϵ/a is small,

$$a = \frac{\lambda}{2l}\left[1 + \left(\frac{a}{l}\right)^2\left\{\frac{\pi}{12}\left(1 + \frac{\pi^2}{32}\frac{\epsilon^2}{a^2}\right) + \frac{\pi^2}{96}\frac{\epsilon^2}{a^2}\right\}\right]$$

$$= \frac{\lambda}{2l}\left[1 + \frac{\pi}{12}\left(1 + \frac{\pi(\pi+4)}{32}\left(\frac{\epsilon}{a}\right)^2\right)\left(\frac{a}{l}\right)^2\right].$$

It is important to notice that the condition here derived is the condition for the vanishing of the central fringe in the diffraction pattern. With white light the fringes are observable only near the centre of the diffraction pattern, and under conditions of good definition

the vanishing of the central fringe is probably the most suitable form in which to make the observation. With poor definition, however, regard will be paid to the minimum visibility of the pattern as a whole rather than to that of the central fringe only.

When $\frac{\pi l\alpha}{\lambda} = \frac{\pi}{2} + kp^2$, the general illumination—in so far as the terms depending upon θ are concerned—will be given by

$$\frac{4}{\pi}\mathrm{I} = -\tfrac{1}{3}p^2 \cdot \frac{\pi^2 l^2}{\lambda^2}\theta^2 - \mathrm{A}_0 \cdot kp^2 \cdot \cos\frac{2\pi l\theta}{\lambda} + \tfrac{1}{6}\mathrm{A}_0 \cdot \frac{\pi^2 l}{\lambda} \cdot \theta \sin\frac{2\pi l\theta}{\lambda} + \tfrac{1}{3}\mathrm{B}_1 \cdot \pi mp^2 \cos\frac{2\pi l\theta}{\lambda}.$$

In general, therefore, apart from a constant factor,

$$\mathrm{I}\left(\theta + \frac{\lambda}{2l}\right) - \mathrm{I}(\theta) = -\tfrac{1}{3}p^2\pi^2\left(\tfrac{1}{4} + \frac{l}{\lambda}\theta\right) + 2\mathrm{A}_0 \cdot kp^2 \cdot \cos\frac{2\pi l\theta}{\lambda}$$
$$- \tfrac{1}{6}\mathrm{A}_0 \cdot \frac{\pi^2 l}{\lambda}\left(\frac{\lambda}{2l} + 2\theta\right)\sin\frac{2\pi l\theta}{\lambda} - \tfrac{1}{3}\mathrm{B}_1 \cdot \pi mp^2 \cos\frac{2\pi l\theta}{\lambda}.$$

This expression represents the difference in the illumination at any two points of the field whose distance apart is equal to the width of one interference fringe. Putting $\theta = 0$, the condition previously obtained for the vanishing of the central fringe is found again.

It will be seen that for this expression to vanish for any given value of θ, the resulting sign of the k so determined will depend upon the particular value of θ used. Thus the condition for the vanishing of the second fringe reduces to $k = -\pi^2/8\mathrm{A}_0$ approximately. It follows, therefore, that if k is not zero, the contrast at some parts of the field is greater and at other parts is less than if k is zero. If, then, regard is paid to the interference pattern as a whole, the best value of k to use is $k = 0$, corresponding to the simple formula

$$\alpha = \frac{\lambda}{2l}.$$

7. *Double Star, consisting of Two Components of Unequal Size or Surface Brightness.*—The results obtained in the previous section are no longer applicable if the two components of the double star are of unequal size or surface brightness.

To discuss this case, we take an origin as before on the line of and midway between the centres. The separation of the stars is denoted by α and their angular diameters by $\epsilon\left(\text{with centre at } -\frac{\alpha}{2}, 0\right)$ and $\mu\left(\text{with centre at } +\frac{\alpha}{2}, 0\right)$ respectively.

The results are similar if the diameters or surface brightness or both are unequal. We consider, therefore, only the case of two stars of equal surface brightness but unequal in diameter.

From equation (5) it is at once seen that the intensity in this case is given by

$$I = \frac{\epsilon^2}{2}\int_{-1}^{+1}(1-u^2)^{\frac{1}{2}}\left[\frac{\sin p\left(mu-\frac{\pi l\theta}{\lambda}-\frac{\pi l\alpha}{2\lambda}\right)}{p\left(mu-\frac{\pi l\theta}{\lambda}-\frac{\pi l\alpha}{2\lambda}\right)}\right]^2\cos^2\left(mu+\frac{\pi l\theta}{\lambda}-\frac{\pi l\alpha}{2\lambda}\right)du$$

$$+\frac{\mu^2}{2}\int_{-1}^{+1}(1-u^2)^{\frac{1}{2}}\left[\frac{\sin p\left(nu-\frac{\pi l\theta}{\lambda}+\frac{\pi l\alpha}{2\lambda}\right)}{p\left(nu-\frac{\pi l\theta}{\lambda}+\frac{\pi l\alpha}{2\lambda}\right)}\right]^2\cos^2\left(nu-\frac{\pi l\theta}{\lambda}-\frac{\pi l\alpha}{2\lambda}\right)du \quad (7)$$

where

$$m=\frac{\pi l\epsilon}{2\lambda}\qquad n=\frac{\pi l\mu}{2\lambda}.$$

Expanding to the second power of p, and writing

$$\int_0^1(1-u^2)^{\frac{1}{2}}\cos 2mu\,du=A_{m,0}\qquad \int_0^1(1-u^2)^{\frac{1}{2}}\cos 2nu\,du=A_{n,0}$$

$$\int_0^1(1-u^2)^{\frac{1}{2}}u^2\cos 2mu\,du=A_{m,1}\qquad \int_0^1(1-u^2)^{\frac{1}{2}}u^2\cos 2nu\,du=A_{n,1}$$

$$\int_0^1(1-u^2)^{\frac{1}{2}}u\sin 2nu\,du=B_{m,1}\qquad \int_0^1(1-u^2)^{\frac{1}{2}}u\sin 2nu\,du=B_{n,1},$$

it is found, on reduction, that

$$I=\text{const.}-\frac{p^2}{3}\cdot\frac{\pi}{16}(m^2\epsilon^2+n^2\mu^2)-\frac{p^2}{3}\cdot\frac{\pi}{4}\cdot\frac{\pi^2l^2}{\lambda^2}\left[\epsilon^2\left(\theta+\frac{\alpha}{2}\right)^2+\mu^2\left(\theta-\frac{\alpha}{2}\right)^2\right]$$

$$+\frac{\pi}{4}\left[A_{m,0}\cdot\epsilon^2\cdot\cos\frac{\pi l}{\lambda}(2\theta+\alpha)+A_{n,0}\mu^2\cdot\cos\frac{\pi l}{\lambda}(2\theta-\alpha)\right]$$

$$-\frac{\pi}{4}\cdot\frac{p^2}{3}\left[A_{m,1}\cdot m^2\epsilon^2\cdot\cos\frac{\pi l}{\lambda}(2\theta+\alpha)+A_{n,1}n^2\mu^2\cos\frac{\pi l}{\lambda}(2\theta-\alpha)\right]$$

$$-\frac{\pi}{4}\cdot\frac{p^2}{3}\left[A_{m,0}\epsilon^2\cdot\frac{\pi^2l^2}{\lambda^2}\left(\theta+\frac{\alpha}{2}\right)^2\cos\frac{\pi l}{\lambda}(2\theta+\alpha)+A_{n,0}\mu^2\cdot\frac{\pi^2l^2}{\lambda^2}\left(\theta-\frac{\alpha}{2}\right)^2\cos\frac{\pi l}{\lambda}(2\theta-\alpha)\right]$$

$$+\frac{\pi}{4}\cdot\frac{2}{3}p^2\left[B_{m,1}\cdot m\epsilon^2\cdot\frac{\pi l}{\lambda}\left(\theta+\frac{\alpha}{2}\right)\sin\frac{\pi l}{\lambda}(2\theta+\alpha)+B_{n,1}n\mu^2\cdot\frac{\pi l}{\lambda}\left(\theta-\frac{\alpha}{2}\right)\sin\frac{\pi l}{\lambda}(2\theta-\alpha)\right] \quad (8)$$

When $p=0$, *i.e.* for narrow slits,

$$2I=\text{const.}+\frac{\pi}{4}\left[A_{m,0}\cdot\epsilon^2\cdot\cos\frac{\pi l}{\lambda}(2\theta+\alpha)+A_{n,0}\mu^2\cdot\cos\frac{\pi l}{\lambda}(2\theta-\alpha)\right].$$

In this case I is a maximum or minimum when

$$\cos\frac{2\pi l\theta}{\lambda}\cdot\sin\frac{\pi l\alpha}{\lambda}\left[A_{m,0}\epsilon^2-A_{n,0}\mu^2\right]+\sin\frac{2\pi l\theta}{\lambda}\cdot\cos\frac{\pi l\alpha}{\lambda}\left[A_{m,0}\epsilon^2+A_{n,0}\mu^2\right]=0.$$

Let $\theta = \phi$ be the least value of θ to satisfy this equation. Then

$$\tan\frac{2\pi l\phi}{\lambda} = \tan\frac{\pi l a}{\lambda}\cdot\frac{\mu^2 A_{n,0} - \epsilon^2 A_{m,0}}{\mu^2 A_{n,0} + \epsilon^2 A_{m,0}}.$$

With this value of θ,

$$2I = \text{const.} + \frac{\pi}{4}\left[\epsilon^4 A_{m,0}{}^2 + 2\epsilon^2\mu^2 A_{m,0}A_{n,0}\cos\frac{2\pi l a}{\lambda} + \mu^4 A_{n,0}{}^2\right]^{\frac{1}{2}} \qquad (9)$$

In this case the component of I, which depends upon θ, cannot be made to vanish for any value of la/λ; in other words, the interference fringes cannot be made to disappear by variation of the distance apart of the slits. But a variation in the relative intensities of the maxima and minima is produced. From (9) it is evident that the contrast becomes a minimum when $2\pi la/\lambda = \pi$ or

$$a = \frac{\lambda}{2l}.$$

This corresponds to the case in which the fringes vanish when the two component stars are equal.

When p is not zero, we require to express the condition that the intensity difference between the central maximum or minimum and the adjacent minimum or maximum should be as small as possible, by appropriate choice of a. We have just seen that for $p = 0$ the required value of a is $\lambda/2l$.

From (8) we find, on reduction, that

$$\begin{aligned}
F(a) \equiv I\left(\phi + \frac{\lambda}{2l}\right) - I(\phi)
&= -2\left[\epsilon^4 A_{m,0}{}^2 + 2\epsilon^2\mu^2 A_{m,0}A_{n,0}\cos\frac{2\pi l a}{\lambda} + \mu^4 A_{n,0}{}^2\right]^{\frac{1}{2}} \\
&\quad - \frac{p^2}{3}\cdot\frac{\pi^2 l}{2\lambda}\left[\epsilon^2\left(2\phi + a + \frac{\lambda}{2l}\right) + \mu^2\left(2\phi - a + \frac{\lambda}{2l}\right)\right] \\
&\qquad + \frac{2p^2}{3}\left[m^2\epsilon^2 A_{m,1}\cos\frac{\pi l}{\lambda}(2\phi + a) + n^2\mu^2 A_{n,1}\cos\frac{\pi l}{\lambda}(2\phi - a)\right] \\
&\quad + \frac{p^2}{3}\cdot\frac{\pi^2 l^2}{\lambda^2}\left[\epsilon^2 A_{m,0}\left\{\left(\phi + \frac{a}{2} + \frac{\lambda}{2l}\right)^2 + \left(\phi + \frac{a}{2}\right)^2\right\}\cos\frac{\pi l}{\lambda}(2\phi + a)\right. \\
&\qquad \left. + \mu^2 A_{n,0}\left\{\left(\phi - \frac{a}{2} + \frac{\lambda}{2l}\right)^2 + \left(\phi - \frac{a}{2}\right)^2\right\}\cos\frac{\pi l}{\lambda}(2\phi - a)\right] \\
&\quad + \frac{2p^2}{3}\cdot\frac{\pi l}{\lambda}\left[m\epsilon^2 B_{m,1}\sin\frac{\pi l}{\lambda}(2\phi + a)\left\{2\phi + a + \frac{\lambda}{2l}\right\}\right. \\
&\qquad \left. + n\mu^2 B_{n,1}\sin\frac{\pi l}{\lambda}(2\phi - a)\left\{2\phi - a + \frac{\lambda}{2l}\right\}\right] \qquad . \quad (10)
\end{aligned}$$

This represents the intensity difference between the central maximum and minimum, since it may be shown exactly as in § 3 that the value of $I(\theta)$, when p is not zero, for the value of θ which makes I a maximum or minimum, and which corresponds to the value θ_0 when $p = 0$, is $I(\theta_0)$.

We assume that $\epsilon > \mu$. Then, when $p = 0$,

$$\frac{4}{\pi} I = \frac{\epsilon^2 + \mu^2}{2} + A_{m,0}\epsilon^2 \cos \frac{\pi l}{\lambda}(2\theta + \alpha) + A_{n,0}\mu^2 \cos \frac{\pi l}{\lambda}(2\theta - \alpha),$$

and when $\alpha = \frac{\lambda}{2l}$,

$$\frac{4}{\pi} I = \frac{\epsilon^2 + \mu^2}{2} - [\epsilon^2 A_{m,0} - \mu^2 A_{n,0}] \sin \frac{2\pi l\theta}{\lambda}.$$

This determines the sign to be given to the radical in $F(\alpha)$. When $\epsilon > \mu$, the value of ϕ corresponding to $\alpha = \frac{\lambda}{2l}$ is $\phi = -\frac{\lambda}{4l}$. Then

$$\frac{4}{\pi}\left\{ I\left(\phi + \frac{\lambda}{2l}\right) - I(\phi) \right\} : \frac{4}{\pi}\left\{ I\left(\frac{\lambda}{4l}\right) - I\left(-\frac{\lambda}{4l}\right) \right\} = -2(\epsilon^2 A_{m,0} - \mu^2 A_{n,0}).$$

When p is finite, we must assume in this case that

$$\frac{\pi l\alpha}{\lambda} = \frac{\pi}{2} + kp \qquad \frac{2\pi l\phi}{\lambda} = -\frac{\pi}{2} + k'p.$$

Then

$$\left[\epsilon^4 A_{m,0}^2 + 2\epsilon^2\mu^2 A_{m,0}A_{n,0} \cos \frac{2\pi l\alpha}{\lambda} + \mu^4 A_{n,0}^2\right]^{\frac{1}{2}}$$

$$= \left[(\epsilon^2 A_{m,0} - \mu^2 A_{n,0})^2 + 4\epsilon^2\mu^2 A_{m,0}A_{n,0}k^2p^2\right]^{\frac{1}{2}}$$

$$= (\epsilon^2 A_{m,0} - \mu^2 A_{n,0})\left[1 + \frac{2\epsilon^2\mu^2 A_{m,0}A_{n,0}}{[\epsilon^2 A_{m,0} - \mu^2 A_{n,0}]^2} k^2p^2\right],$$

the sign being chosen to agree with the value when $p = 0$.

In the terms in $F(\alpha)$ containing p^2 we can put $\alpha = \frac{\lambda}{2l}$, $2\phi = -\frac{\lambda}{2l}$. Then, using (10),

$$F(\alpha) = -2(\epsilon^2 A_{m,0} - \mu^2 A_{n,0}) - 2\frac{\epsilon^2\mu^2 A_{m,0}A_{n,0}}{[\epsilon^2 A_{m,0} - \mu^2 A_{n,0}]^2}k^2p^2$$

$$-\frac{p^2}{3}\cdot\frac{\pi^2}{4}(\epsilon^2 - \mu^2) + \frac{p^2}{3}\cdot\frac{\pi^2}{4}[^2A_{m,0} - \mu^2 A_{n,0}] + \frac{2p^2}{3}[m^2\epsilon^2 A_{m,1} - n^2\mu^2 A_{n\,1}].$$

Since m, n are small, being proportional to ϵ/α, μ/α, we may put

$$A_{m,0} = 1 - \tfrac{1}{2}m^2 \qquad A_{n,0} = 1 - \tfrac{1}{2}n^2$$
$$A_{m,1} = \tfrac{1}{4} - \tfrac{1}{2}m^2 \qquad A_{n,1} = \tfrac{1}{4} - \tfrac{1}{2}n^2.$$

The coefficient of p^2 in the above expression is then

$$-\frac{2\epsilon^2\mu^2 A_{m,0}A_{n,0}}{(\epsilon^2 A_{m,0} - \mu^2 A_{n,0})^2}k^2 - \frac{\pi^2}{12}\cdot\frac{m^2\epsilon^2 - n^2\mu^2}{2} + \tfrac{1}{6}(m^2\epsilon^2 - u^2n^2),$$

and finally

$$F(\alpha) = -2(\epsilon^2 A_{m,0} - \mu^2 A_{n,0}) - p^2\left[k^2 \cdot \frac{2\epsilon^2\mu^2 A_{m,0} A_{n,0}}{(\epsilon^2 A_{m,0} - \mu^2 A_{n,0})^2} + \frac{(\pi^2-4)\pi^2}{3 \cdot 2^7} \cdot \frac{\epsilon^4 - \mu^4}{a^2}\right],$$

where we have substituted

$$m = \frac{\pi}{4}\frac{\epsilon}{a} \qquad n = \frac{\pi}{4}\frac{\mu}{a}.$$

The coefficient of p^2 within the brackets is positive for all values of k. The two terms in $F(\alpha)$ have therefore always the same sign, and it is evident that the inclusion of the k term serves only to increase the contrast. In other words, when the apertures have finite width the minimum contrast is still obtained when their distance apart is given by

$$\alpha = \frac{\lambda}{2l}.$$

This result does not hold if the two stars are equal or nearly equal, as then $(\epsilon^2 A_{m,0} - \mu^2 A_{n,0})$ vanishes. The results obtained in the previous section are then applicable.

8. *The Method of Rotating Apertures.*—For the observation of double stars with the interferometer, instead of placing the slits at right angles to the line joining the centres of the stars and varying their separation, it is usually more convenient to keep the separation constant and to rotate the slits. The phenomena to be expected under these circumstances will now be examined.

We take, as before, an origin midway between the slits and the axes Oy, Oz in the plane of and respectively perpendicular and parallel to the slits. When the axis Oy makes an angle ξ with the line of centres of the two stars, the co-ordinates of the centres are respectively $\left(\frac{a}{2}\cos\xi, \frac{a}{2}\sin\xi\right)$ and $\left(-\frac{a}{2}\cos\xi, -\frac{a}{2}\sin\xi\right)$.

We assume first that the two stars are of equal diameter and brightness. Then the equations of their contours are

$$\left(\psi \mp \frac{a}{2}\sin\xi\right)^2 + \left(\omega \mp \frac{a}{2}\cos\xi\right)^2 = \frac{\epsilon^2}{4},$$

where like signs are to be taken together.

Then, considering one star only, equation (2) gives

$$I = \int d\omega \int_{\psi_1}^{\psi_2} \left[\frac{\sin\frac{\pi a}{\lambda}(\omega-\theta)}{\frac{\pi a}{\lambda}(\omega-\theta)}\right]^2 \cos^2\frac{\pi l}{\lambda}(\omega-\theta)\,d\psi$$

$$= \int_{-\frac{\epsilon}{2}+\frac{a}{2}\cos\xi}^{+\frac{\epsilon}{2}+\frac{a}{2}\cos\xi} 2\left[\frac{\epsilon^2}{4} - \left(\omega - \frac{a}{2}\cos\xi\right)^2\right]^{\frac{1}{2}} \left[\frac{\sin\frac{\pi a}{\lambda}(\omega-\theta)}{\frac{\pi a}{\lambda}(\omega-\theta)}\right]^2 \cos^2\frac{\pi l}{\lambda}(\omega-\theta)\,d\omega.$$

Putting

$$\omega - \frac{a}{2}\cos\xi = \frac{\epsilon}{2}.u,$$

$$I = \frac{\epsilon^2}{2}\int_{-1}^{+1}(1-u^2)^{\frac{1}{2}}.\left[\frac{\sin p\left(mu - \frac{\pi l\theta}{\lambda} + \frac{\pi l}{2\lambda}a\cos\xi\right)}{p\left(mu - \frac{\pi l\theta}{\lambda} + \frac{\pi l}{2\lambda}a\cos\xi\right)}\right]^2 \cos^2\left(mu - \frac{\pi l\theta}{\lambda} + \frac{\pi l}{2\lambda}a\cos\xi\right)d\omega.$$

This is analogous to the second term on the right-hand side of equation (5), except that $a\cos\xi$ replaces a. The argument may therefore be developed exactly as in the case when $\xi = 0$, leading to the conclusion that when $p = 0$ the interference fringes will disappear, provided that

$$a\cos\xi = \frac{\lambda}{2l}.$$

In deriving this result, however, the term containing ϕ in equation (1) has been neglected. In the present case it is not legitimate to do this, since the two stars being separated relatively to one another parallel to the slits, the points of maximum intensity of the interference patterns produced by the two stars are displaced relatively to one another parallel to the Oz axis.

The integration cannot be performed explicitly when the term in ϕ is included, but it is possible to investigate in a general way the nature of the resulting phenomena. The factor may be taken outside the limits of the integration, and equation (1) may then be rewritten in the form

$$I = \left[\frac{\sin\frac{\pi h}{\lambda}(\phi - \psi')}{\frac{\pi h}{\lambda}(\phi - \psi')}\right]^2 \iint J^2 \left[\frac{\sin\frac{\pi a}{\lambda}(\omega - \theta)}{\frac{\pi a}{\lambda}(\omega - \theta)}\right]^2 \cos^2\frac{\pi l}{\lambda}(\omega - \theta)d\omega\, d\psi,$$

where ψ' denotes a value of ψ included within the range of integration $-\frac{\epsilon}{2}$ to $+\frac{\epsilon}{2}$.

The factor in ϕ has a maximum value unity when $\phi = \psi'$ and is an even function in $(\phi - \psi')$. Since we require only to consider a small range of values of ϕ, its effect can be estimated, without serious error, by replacing the factor by $1 - k\left(\phi - \frac{a}{2}\sin\xi\right)^2$. This assumes that the intensity is a maximum at the centre of the rise and falls off within the region considered proportionally to the square of the distance from the centre.

Then in the case of two equal stars and narrow slits we have approximately

$$I = \text{const.} + A_0\left[1 - k\left(\phi - \frac{a}{2}\sin\xi\right)^2\right]\cos\frac{\pi l}{\lambda}\left(2\theta - a\cos\xi\right)$$
$$+ A_0\left[1 - k\left(\phi + \frac{a}{2}\sin\xi\right)^2\right]\cos\frac{\pi l}{\lambda}\left(2\theta + a\cos\xi\right) \quad . \quad (11)$$

At the centre of the field $\phi = 0$, and the minimum contrast is obtained when $\xi = \xi_0$, given by

$$\alpha \cos \xi_0 = \frac{\lambda}{2l}.$$

If the slits are rotated through a slight angle, so that $\xi = \xi_0 + \delta\xi$, we find approximately

$$I = \text{const.} + 2A_0\alpha \sin \xi_0 \left[\phi k \sin \frac{2\pi l\theta}{\lambda} + \frac{\pi l}{\lambda} \delta\xi \cos \frac{2\pi l\theta}{\lambda} \right]$$

$$+ 2A_0 . \alpha \cos \xi_0 . \phi k \delta\xi \sin \frac{2\pi l\theta}{\lambda}.$$

The last term can be neglected, being of the second order.

When $\alpha \cos \xi_0 = \frac{\lambda}{2l}$, the bright fringes due to one star are superposed upon the dark fringes due to the other star. The central fringes are at $\theta = \pm \frac{\lambda}{4l}$.

When $\theta = +\frac{\lambda}{4l}$ $\quad I = c + 2A_0\alpha \sin \xi_0 . \phi k$

„ $\theta = 0$ $\quad I = c + 2A_0\alpha \sin \xi_0 . \frac{\pi l}{\lambda} \delta\xi$

„ $\theta = -\frac{\lambda}{4l}$ $\quad I = c - 2A_0\alpha \sin \xi_0 . \phi k.$

When $\delta\xi = 0$ $I = c$. We therefore see that the field will be bright for $\theta = +\frac{\lambda}{4l}$ when ϕ is positive, and for $\theta = -\frac{\lambda}{4l}$ when ϕ is negative. It is also bright for $\theta = 0$ when $\delta\xi$ is positive.

Thus when ξ has a value slightly greater than the value for which the fringes disappear, the general appearance near the centre of the field will be that of a bright fringe passing from above and from the right of the centre, through the origin to below and to the left of the centre.

When $\xi = \xi_0 - \delta\xi$ we have, on the other hand,

For $\theta = +\frac{\lambda}{4l}$ $\quad I = c + 2A_0\alpha \sin \xi_0 . \phi k$

„ $\theta = 0$ $\quad I = c - 2A_0\alpha \sin \xi_0 . \frac{\pi l}{\lambda} \delta\xi$

„ $\theta = -\frac{\lambda}{4l}$ $\quad I = c - 2A_0\alpha \sin \xi_0 . \phi k.$

The centre of the field is relatively dark. The field is also dark for $\theta = +\frac{\lambda}{4l}$, ϕ negative, and for $\theta = -\frac{\lambda}{4l}$, ϕ positive. In this case, therefore, a dark fringe will be seen passing from above and from the left of the centre, through the origin to below and to the right of the centre.

The most convenient method of making the observation is therefore to observe the two values of ξ_0, ξ_1, and ξ_2 such that in the one case the oblique bright fringe and in the other case the oblique dark fringe can just be seen. The mean value $\xi_0 = \frac{1}{2}(\xi_1 + \xi_2)$ satisfies the equation $\alpha \cos \xi_0 = \frac{\lambda}{4l}$. A value of ξ_0 can be found in each quadrant, and the mean value of all the results should be used. This method has been employed by Anderson at Mount Wilson (*Astroph. Journ.*, **55**, 48, 1922).

9. *Method of Rotating Apertures for Finite Width of Aperture.*— If the width of slit cannot be neglected, it is desirable to ascertain whether the preceding results require modification. It will be sufficient to place A_0 equal to unity and to neglect terms containing m or m^2.

Then equation (11) must be replaced by the following, which is easily derived from (6),

$$\begin{aligned} I = \text{const.} + \left[1 - k\left(\phi - \frac{\alpha}{2}\sin\xi\right)^2\right]\left[\cos\frac{\pi l}{\lambda}(2\theta - \alpha\cos\xi) - \frac{p^2}{3}\cdot\frac{\pi^2 l^2}{\lambda^2}\left(\theta - \frac{\alpha}{2}\cos\xi\right)^2\right. \\ \left. - \frac{p^2}{3}\cdot\frac{\pi^2 l^2}{\lambda^2}\left(\theta - \frac{\alpha}{2}\cos\xi\right)^2 \cos\frac{\pi l}{\lambda}(2\theta - \alpha\cos\xi)\right] \\ + \left[1 - k\left(\phi + \frac{\alpha}{2}\sin\xi\right)^2\right]\left[\cos\frac{\pi l}{\lambda}(2\theta + \alpha\cos\xi) - \frac{p^2}{3}\cdot\frac{\pi^2 l^2}{\lambda^2}\left(\theta + \frac{\alpha}{2}\cos\xi\right)^2\right. \\ \left. - \frac{p^2}{3}\cdot\frac{\pi^2 l^2}{\lambda^2}\left(\theta + \frac{\alpha}{2}\cos\xi\right)^2 \cdot \cos\frac{\pi l}{\lambda}(2\theta + \alpha\cos\xi)\right]. \end{aligned}$$

$\theta = \frac{\alpha}{2}\cos\xi$ is still the first maximum for the right-hand star, and $\theta = -\frac{\alpha}{2}\cos\xi$ that for the left-hand star.

For $\xi = \xi_0 + \delta\xi$ we get approximately,

$$\begin{aligned} I = \text{const.} + \left[1 - k\left(\phi - \frac{\alpha}{2}\sin\xi_0\right)^2\right]\left[\sin\frac{2\pi l\theta}{\lambda} - \frac{2p^2}{3}\cdot\frac{\pi^2 l^2}{\lambda^2}\left(\theta - \frac{\lambda}{4l}\right)^2 \cos\frac{2\pi l}{\lambda}\left(\theta - \frac{\lambda}{4l}\right)\right. \\ \left. + \cos\frac{2\pi l\theta}{\lambda}\cdot\frac{\pi l}{\lambda}\alpha\sin\xi_0\delta\xi\right] \\ + \left[1 - k\left(\phi + \frac{\alpha}{2}\sin\xi_0\right)^2\right]\left[-\sin\frac{2\pi l\theta}{\lambda} - \frac{2p^2}{3}\cdot\frac{\pi^2 l^2}{\lambda^2}\left(\theta + \frac{\lambda}{4l}\right)^2 \cos\frac{2\pi l}{\lambda}\left(\theta - \frac{\lambda}{4l}\right)\right. \\ \left. + \cos\frac{2\pi l\theta}{\lambda}\cdot\frac{\pi l}{\lambda}\cdot\alpha\sin\xi_0\delta\xi\right]. \end{aligned}$$

Then we have, when

$$\theta = +\frac{\lambda}{4l} \qquad I = c + 2a\sin\xi_0 \cdot \phi k$$

$$\theta = 0 \qquad I = c + 2a\sin\xi_0 \cdot \frac{\pi l}{\lambda}\delta\xi - \frac{p^2\pi^2}{24}$$

$$\theta = -\frac{\lambda}{4l} \qquad I = c - 2a\sin\xi_0 \cdot \phi k.$$

Therefore, as before, when $\delta\xi$ is positive the field is bright above and to the right of the centre and below and to the left of the centre. There is a decrease in intensity at the centre as compared with the case of narrow apertures.

When $\delta\xi$ is negative the field is dark above and to the left of the centre and also below and to the right of the centre. At the centre the field is darker than in the case of narrow apertures.

The general appearance is conditioned mainly by the phenomena just to right and left of the centre, so that the same method can be applied, and there is evidently no correction required to be applied to the formula $a\cos\xi_0 = \frac{\lambda}{2l}$ to allow for a finite width of aperture. Anderson (*loc. cit.*) has verified this experimentally.

10. *Method of Rotating Apertures for Unequal Stars.*—In the case of unequal stars we need consider only the case of narrow apertures and of diameters relatively small compared with the separation of the components. Then $p=0$, $A_0=1$, $A_1=1$, $B_1=0$.

Then

$$I = \text{const.} + \epsilon^2\left[1 - k\left(\phi - \frac{a}{2}\sin\xi\right)^2\right]\cos\frac{\pi l}{\lambda}(2\theta - a\cos\xi) + \mu^2\left[1 - k\left(\phi + \frac{a}{2}\cos\xi\right)^2\right]\cos\frac{\pi l}{\lambda}(2\theta + a\cos\xi).$$

It follows, as in § 7, that the minimum contrast at $\phi=0$ is given by $a\cos\xi_0 = \frac{\lambda}{2l}$.

For this value of ξ,

$$I = c + \epsilon^2\left[1 - k\left(\phi - \frac{a}{2}\sin\xi_0\right)^2\right]\sin\frac{2\pi l\theta}{\lambda} - \mu^2\left[1 - k\left(\phi + \frac{a}{2}\sin\xi_0\right)^2\right]\sin\frac{2\pi l\theta}{\lambda}.$$

If $\xi = \xi_0 + \delta\xi$, we obtain

$$I = c + \epsilon^2\left[1 - k\left(\phi - \frac{a}{2}\sin\xi_0\right)^2\right]\left[\sin\frac{2\pi l\theta}{\lambda} + \cos\frac{2\pi l\theta}{\lambda}\cdot\frac{\pi l a}{\lambda}\sin\xi_0\delta\xi\right] + \mu^2\left[1 - k\left(\phi + \frac{a}{2}\sin\xi_0\right)^2\right]\left[-\sin\frac{2\pi l\theta}{\lambda} + \cos\frac{2\pi l\theta}{\lambda}\cdot\frac{\pi l a}{\lambda}\sin\xi_0\delta\xi\right].$$

We consider the intensities at $\theta = \pm\frac{\lambda}{4l}$, $\phi = \pm\frac{a}{2}\sin\xi_0$ (the centres of the two discs).

	$\phi = +\frac{a}{2}\sin\xi_0$	$\phi = -\frac{a}{2}\sin\xi_0$
$\theta = +\frac{\lambda}{4l}$	$I = c + (\epsilon^2 - \mu^2) + k\mu^2 a^2\sin^2\xi_0$	$I = c + (\epsilon^2 - \mu^2) - k\epsilon^2 . a^2\sin^2\xi_0$
$\theta = 0$	$= c + \frac{\pi l a}{\lambda}(\epsilon^2 + \mu^2)\sin\xi_0\delta\xi$	$= c + \frac{\pi l a}{\lambda}(\epsilon^2 + \mu^2)\sin\xi_0\delta\xi$
$\theta = -\frac{\lambda}{4l}$	$= c - (\epsilon^2 - \mu^2) + k\epsilon^2 a^2\sin^2\xi_0$	$= c - (\epsilon^2 - \mu^2) - k\mu^2 . a^2\sin^2\xi_0$.

The terms in $(\epsilon^2 - \mu^2)$ represent a uniform change in intensity across the field at the centre, the intensity increasing towards the right or the left according as $\epsilon >$ or $< \mu$. Superposed upon this field of intensity is an additional field which presents exactly the same characteristics as the field studied in § 8. When the stars are not equal, therefore, the method of rotating apertures can still be used, but the bright and dark fringes at the centre will be superposed upon a field whose intensity varies uniformly from right to left in the region under examination.

11. *Conclusions.*—The results obtained in the present investigation may be summed up as follows :—

(i) In the interferometer measures of the angular diameter of a star, the interference fringes vanish for a separation, l, of the apertures such that $\alpha = 1{\cdot}22\lambda/l$, α being the angular diameter of the star. The star is assumed to be circular and of uniform brightness.

(ii) If the star is not circular, but elliptical, and of uniform brightness, the same result applies, provided that α is taken to be the angular diameter measured in a direction at right angles to the slits.

(iii) If the star shows darkening towards the limb, a constant greater than 1·22 is required in the above formula. The particular value of the constant for any law of darkening can be obtained by quadratures.

(iv) If the width of the slits, a, is not small compared with their separation, l, the formula becomes

$$\alpha = 1{\cdot}22\frac{\lambda}{l}\left[1 + 0{\cdot}765\left(\frac{a}{l}\right)^2\right].$$

(v) In the case of a double star with components of equal size and brightness and for narrow slits, the fringes vanish at the centre of the field when $\alpha = \dfrac{\lambda}{2l}$, α being the angular separation.

(vi) If the apertures have finite width and the observation is made on the vanishing of the central fringe, this formula becomes

$$\alpha = \frac{\lambda}{2l}\left[1 + 0{\cdot}262\left(\frac{a}{l}\right)^2\right].$$

But if, as is usually the case with poor definition, regard is had to the visibility of the interference pattern as a whole, the simple formula $\alpha = \dfrac{\lambda}{2l}$ must be used.

The correction to be applied if the ratio of the angular diameters of the components to their separation is not negligible is indicated.

(vii) When the two components are of unequal size or brightness, the interference pattern at the centre never completely vanishes, but the contrast becomes a minimum when $\alpha = \dfrac{\lambda}{2l}$. This result remains unaltered for finite width of aperture.

(viii) For the measurement of the angular separation of double stars, the method of rotating apertures is the most accurate. If ξ is the angle between the line of centres and the direction at right angles to the slits

when the central fringe changes from an oblique bright fringe to an oblique dark fringe for small change in ξ,

$$a \cos \xi = \frac{\lambda}{2l}.$$

This result holds also for finite width of aperture. Its application in the case of unequal stars is discussed.

Christmas Island:
1922 *June* 15.

The Sun and Sunspots, 1820–1920. By E. Walter Maunder. (Plates 13, 14, 15, 16.)

(An Address given on the evening of Monday, 1922 May 29, at the Conversazione held in connection with the Centenary of the Royal Astronomical Society.)

The Royal Astronomical Society was founded in 1820. At that date it was known that there was a Sun, that sometimes there were spots upon its surface, and that the Sun rotated on its axis. Practically that was all.

In 1826, the systematic study of the Sun's surface was commenced by Schwabe, and it has been continued up to the present time. Schwabe presented his drawings to the Royal Astronomical Society, and as it also possesses those of some of his predecessors in the same field, this Society now holds records of one kind or another showing the changes that have taken place upon the solar surface from the year 1820, and so continuously to the present date.

The question before us this evening is: what views do we now hold of the constitution of the Sun and the relationships of its spots, and upon what facts do we base them?

A quarter of an hour is a short time in which to review the scientific evolution of a century, even when the inquiry is restricted to a single department of astronomy, so confined that it deals only with sunspots as drawn at the telescope, or impressed upon the photograph plate.

My first illustration (Plate 13) deals with the annual percentages of spotless days, 1826–1913, and appeared in the *Monthly Notices*, 74, Plate 4, between pages 114 and 115. It is based upon Schwabe's persistent daily count of sunspots. In itself, his work was very simple and straightforward, but it was carried on systematically and with the utmost patience. These qualities made it great and epoch-making, and they were acknowledged by the award to him, in 1857, of the Gold Medal of this Society.

The diagram is deduced, partly from the observations of Schwabe, and partly from observations subsequent to him. It exhibits for each year from 1826 to 1913 the percentage of the days of observation upon which the Sun appeared to be free from spots. It shows clearly that all years are not equally prolific as to sunspots. Confining ourselves

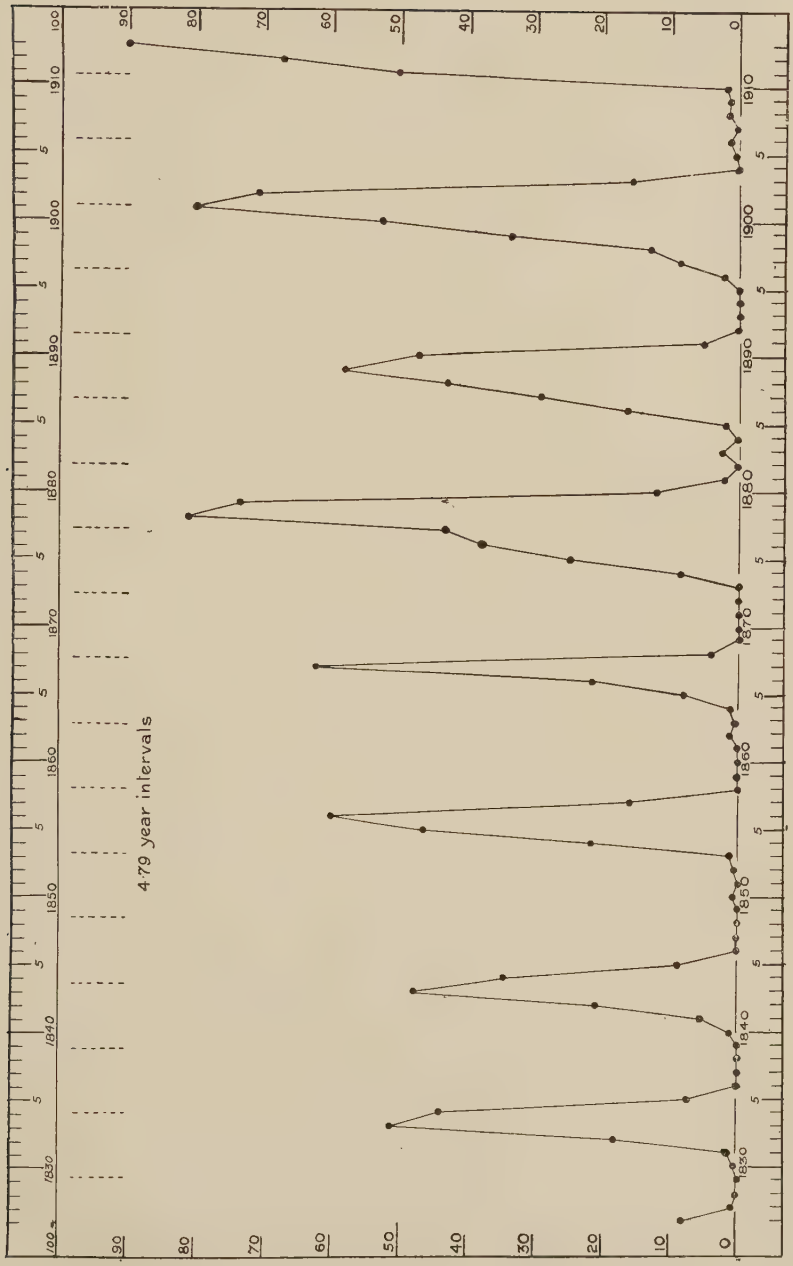

Fig. 1.—Annual Percentages of Spotless Days, 1826–1913

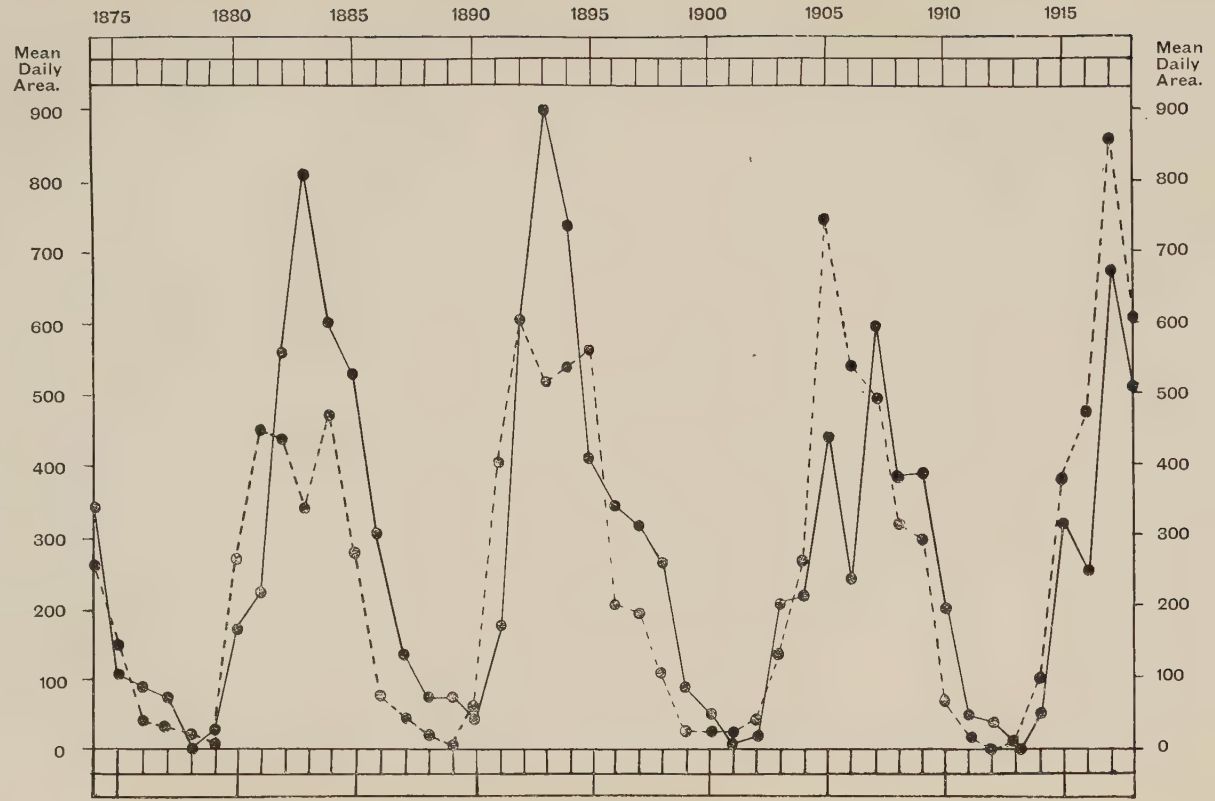

Fig. 2.—Mean Daily Spotted Area, North and South, 1874–1918.
Northern - - - - Southern ———

to the years represented, it appears that there were eight epochs when there was a most marked absence of spots. The dates of such absences are very clearly defined. In the eighty-eight years represented there is in no case any doubt as to the year of the greatest solar quiescence, and a simple examination of the curve demonstrates that the average length of the interval, from one year of greatest quiescence to the next, was eleven and a half years, and that the range in length of that interval was from ten to thirteen years.

This interval is often spoken of as "the Sunspot Period." The use of the word "Period" has been unfortunate, for it has given rise to an impression which is not in accord with the facts of the case. Let us speak of it in future as "the Sunspot Cycle." A series of events may be, and often is, "cyclical" without being in the usual mathematical sense "periodic"; but we are not, at present, justified in applying the term "period," in its strictest mathematical sense, to the interval that occurs between one sunspot minimum and the next.

The second illustration (Plate 14) gives a somewhat fuller study, derived from the measures of the areas of sunspots made at Greenwich Observatory on the photographs of the Sun, taken during the years 1874–1918: it is an extension of a similar diagram given in the *Monthly Notices*, **64**, page 748. It exhibits the mean daily spotted area of the Sun; the total mean spotted area of the northern hemisphere being shown separately from that of the southern. The principal feature of the diagram therefore is the comparison of the sunspot areas in the two hemispheres. From this point of view the diagram is worth careful consideration. Just as the first illustration showed that during the years represented there was only one sunspot cycle—a cycle having a mean duration of about eleven and a half years—so the present diagram shows that the cycle holds good for each hemisphere, as well as for both together. The solar cycle is one. There is no question of the existence of submultiple cycles, or of incommensurable cycles of greater length.

Further examination of the diagram shows that the maximum activity in any particular cycle does not necessarily fall at the same epoch for the two hemispheres; rather, a divergency appears between them. In each of the four cycles represented, the northern activity was developed before the southern; in each of them the curve in one hemisphere attained a single crest, while in the other it attained a double one; or, to put it in other words, the form of the curve was not the same in the two hemispheres. We have therefore here a solar dichotomy displayed. Had the results of the whole century been presented in a like form, other peculiarities would have been seen, but these would have strengthened, not weakened, the case for the solar dichotomy. The solar cycle is one; its manifestations in the two hemispheres, north and south, are different.

The third diagram (Plate 15) appeared in the *Monthly Notices*, **74**, Plate 3, between pages 114 and 115, and brings out another relation. The solar dichotomy is not illustrated here; we are dealing with the whole of the Sun's disc as we see it. The points set forth are the variations in the area covered by sunspots, and the variations in the mean distance

from the equator of the Sun of all spots. Area and latitude are the two factors brought into notice. The period covered by the diagram is 1854–1912; if 1913 had been included, five complete cycles would have been shown. The statistics for the first two cycles are derived from the work of the late Professor Spoerer; for the last three, from the Greenwich Photoheliographic Results.

The continuous line shows the variation in the total daily spotted area of the Sun during the progress of each cycle; the dotted line shows the changing distance from the solar equator of the centre of gravity of the sunspot zones. In this latter curve, we have the fact that the solar cycle is one, and one only, brought out in a most unmistakable fashion: each cycle begins with an activity in high latitude; each cycle ends with the last remnants of activity transferred to a low one. This is what is known as "Spoerer's Law of Zones." Broadly speaking, we may say that the approach toward the equator is continuous from the beginning of the cycle to its end. The distinction between the forms of the two curves is emphatic. The area curve is continuous; it begins practically at zero; it increases up to a certain point; it then diminishes again to the next zero; and so on. Flow and ebb, flow and ebb, follow each other continuously. But the latitude curve is discontinuous. It always starts from high latitudes; when it has reached low latitudes it is cut off; there is no gradual return to the high latitudes. The reappearance of spots in high latitudes is sudden.

There are now three facts before us. There is one solar cycle, shown differently, in time and form of the curve, in the two hemispheres, and now in two manifestations of quite separate character; one in area, a complete pulsation, and one in latitude, which moves in one direction only, *i.e.* from high solar latitudes to low.

The fourth illustration (Plate 16) appeared in the *Monthly Notices*, **74**, Plate 2, between pages 114 and 115. It deals only with the distribution of spots in latitude, nothing being shown concerning the areas of spots. The materials are drawn entirely from the Greenwich Photoheliographic Results, and extend over the years 1874–1913. Seven thousand spots are represented, but the short straight line which indicates the latitude of a spot is drawn just as long and as heavy for a small spot as for a large one, for one spot as for a dozen. All spots occurring during the same rotation of the Sun are represented in the same vertical line. The diagram is concerned only with the distribution of spot centres in heliographic latitude.

This diagram has been familiarly called "the Butterfly Diagram," as it seems to suggest three butterflies pinned down to a board with their wings extended. Heads, bodies, and legs have disappeared, but the outstretched wings remain. Each pair of wings is distinct from the next; there is a clear V-shaped gap between each of the two specimens. Here again the first deduction is reinforced from an altogether different set of facts. The solar cycle is one.

This diagram further suggests that the *origin* of the solar spots lies within the Sun, not without. They come from below the surface; they are not impressed upon the surface by some exterior influence; neither by planets, nor by meteors. No exterior influence could invariably

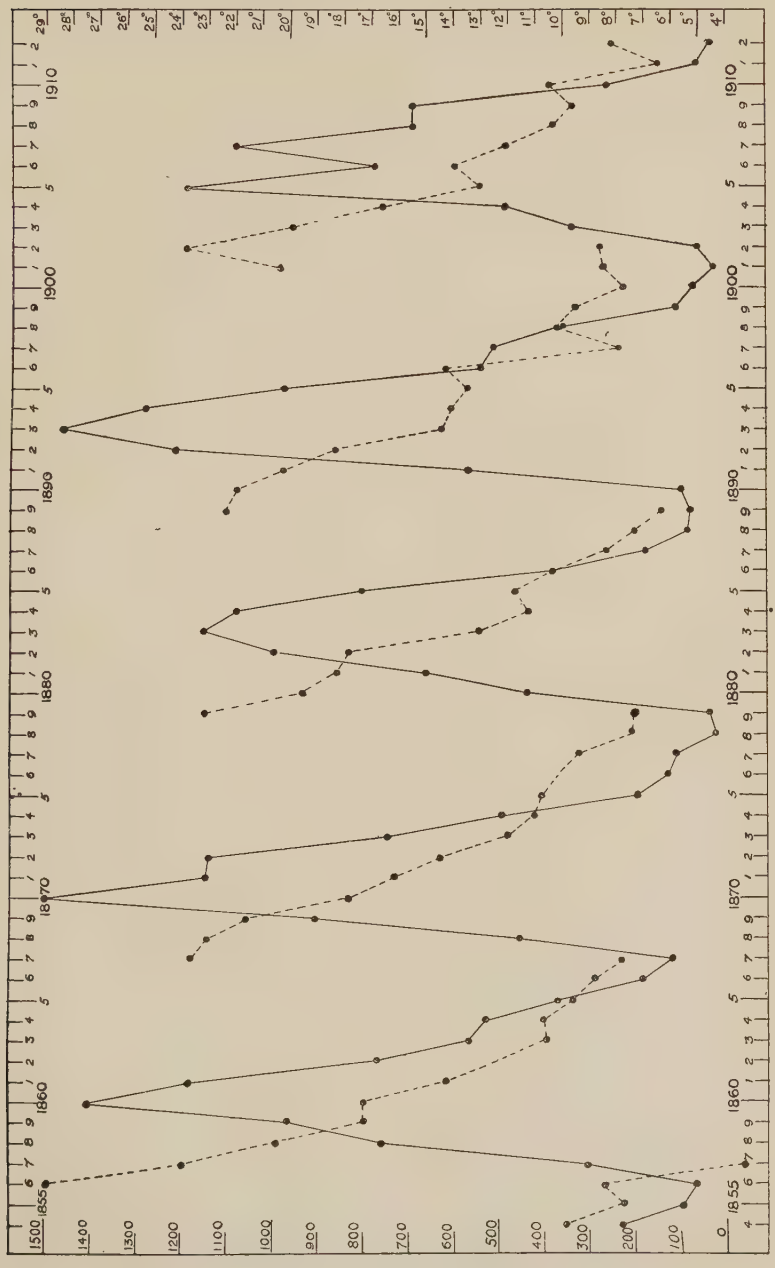

Fig. 3.—Mean Latitude and Mean Spotted Area, 1854-1912. Latitude - - - - Area ———.

begin a fresh disturbance in a high latitude simultaneously on both sides of the equator.

If we take a card with a narrow slit in it, parallel to the Sun's equator, and move it up and down over the diagram, then, wherever that slit is placed within the range of the sunspot zones, the three sunspot cycles will be brought out clearly and unmistakably. Our first conclusion, that the sunspot cycle is one, is now extended; it is true not merely for sunspots in general, but for the spots of any special zone in particular. This conclusion goes deeper than our first, which was merely that the sunspot cycle, on the average, lasted for about eleven and a half years. The Sun as a whole is under the law of that cycle, and each individual zone has its own particular part to play in it. But the Sun as a whole is a unit, no matter how distinct its dichotomy into two hemispheres, no matter how distinct may be the action in any particular zone.

Another feature is suggested by this diagram, but will be better brought out in a different connection. It is that the southern hemisphere appears to encroach slightly on the northern at the equator, or at about the time of the close of one cycle and the beginning of the next. The southern influence seems to cross the equator.

Since the origin of the solar spots lies within the Sun, and the northern and southern spots show differences in their behaviour, we must conclude that the Sun is not symmetrical in the constitution of its interior. If then we assume, as the basis of any investigation, that the Sun is symmetrical in its internal constitution, we are making an assumption contrary to the evidence supplied by the behaviour of its surface.

If there be this clear distinction between the two hemispheres, is it possible that one of them might go out of action for a time, and if so, what would follow?

This is not a mere oratorical question; the event supposed has actually occurred. Dr. Rudolf Wolf, of Zurich, demonstrated, and Spoerer developed that demonstration further, that in the latter part of the seventeenth century and the beginning of the eighteenth the northern hemisphere of the sun failed for many years to produce a single recorded spot.

Was there any effect during that same period recognised here on the earth that could be plausibly associated with this sterility in the Sun's northern hemisphere?

It is suggestive that, while the northern hemisphere was thus entirely barren, the southern hemisphere of the Sun had some spots, but only few, and for nearly seventy years the Sun showed a prolonged spot minimum. The failure in spot-activity of the northern hemisphere was not compensated by an increased activity in the southern. Further, as Miss Agnes Clerke pointed out in *Knowledge* for 1894 September, no auroræ were reported during these years; from which we may infer that there were no great magnetic disturbances taking place on the earth. It is also suggestive that Professor A. E. Douglass, who has been studying with great particularity, the annual rings of trees in relation to climate and the solar activity (*Climatic Cycles and Tree Growths*) found that this same period "was the interval in the yellow pines" (*i.e.* of Northern

Arizona) "which gave me more trouble than any other in trying to work out the action of the sunspot cycle" (*Journal of the British Astronomical Association*, 32, 223). Professor Douglass added that it seemed to him "a very important corroboration of the relationship between solar activity and terrestrial conditions, for I presume that these tree variations are related directly to the weather."

Sunspots, if looked at casually, appear to differ widely in their form and behaviour, but a little systematic observation shows that, for the most part, they conform in their history to a single type. As an illustration we may take the group which attained its maximum development in 1920 March, and of which the Rev. A. L. Cortie has summarised the life-history in the *Monthly Notices*, 80, 574–578, where it is accompanied by seven drawings of its appearance. This type is that of a more or less regular stream, of which the first and last spots are usually the largest, best defined, and the most stable. The straight line joining the two chief spots—sometimes distinguished as the "leader" and "trailer"—was in this case parallel to the Sun's equator.

In streams such as these, in which the dominance of the two chief members is well marked, Professor Hale has pointed out that the leader and trailer are usually of opposite magnetic polarity; and that the magnetic polarity of the two is reversed for opposite sides of the solar equator. Further, it would appear that there has been a reversal of sign of the polarities since the commencement of the present sunspot cycle; that is to say, during the cycle ending in 1913 the leader spots were in the northern hemisphere of minus sign (corresponding to the N. magnetic pole of the earth) and of plus sign in the southern hemisphere; during the present cycle these signs have been reversed.

Professor Hale in the same paper ("Contribution from Mount Wilson Solar Observatory," No. 165), points out that the inclination of the axis of the spot stream, which is in general represented by the line joining the "leader" and the "trailer," bears a definite relation to the latitude at which it occurs. The "trailer" tends to be further from the equator than the "leader," and "in low latitudes the axes are nearly parallel to the Sun's equator; but with increasing latitude the mean inclination increases to a maximum of about 11°" in the region of latitude 30° to 35°.

To go back in the history nearly sixty years, one of the most notable contributions to the answer to the question "What is a Sun?" was supplied by the Redhill Observations of Sunspots made by R. C. Carrington in the years 1853–1861, in which he redetermined the position of the Sun's axis, and the rotation period of the Sun. In particular, he showed that each different zone of latitude, north and south, has its own rotation period. The annexed diagram, fig. 5, will roughly illustrate the general effect. It is drawn on the assumption that there is a spot in every fifth degree of latitude, and that at a given moment these seventeen spots were all observed to be on the central meridian of the Sun's apparent disc. If, then, each spot travelled westward with the *average* speed of apparent motion appropriate to its own particular latitude, then in twenty-seven and one-third days the spots would be found in the positions indicated on the curved line.

Referring again to "the Butterfly Diagram," the shift of the habitat of spot-groups during the progress of the solar cycle shown therein would appear to be a sort of surface-ripple on the Sun, moving in both hemispheres from high latitudes to low, as each cycle proceeds from its beginning to its close. The case of the Cepheid Variables has made us familiar in recent years with the idea of stars which change their volume—stars with pulsating photospheres. In the Sun we have, as yet, no proof of a *general* pulsation of the photosphere; only of this ripple of disturbance on its surface. But, since the rotation period, as

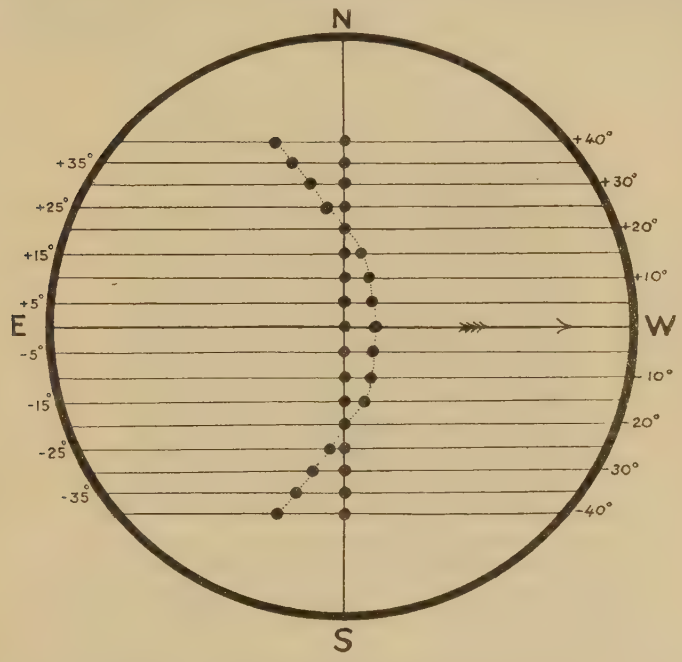

Fig. 5.—To illustrate the relative apparent movements in Longitude of Spots in different Solar Latitudes in one rotation of the Sun.

indicated by sunspots, is shorter as the equator is approached, it follows that the mean rotation period of the Sun is quicker before minimum than after.

But the above diagram puts the fact of the difference in the rotation periods of different latitudes in much too crude a form. A paper on "The Solar Rotation Period from Greenwich Sunspot Measures, 1879–1901," appeared in the *Monthly Notices*, **65**, and on p. 817 a table was given—Table II.—which was intended to show how wide are the differences of rotation period, as derived from different spot groups even in the same latitude. Every group has a proper motion of its own; even within the same group there are internal movements. The leader spot in the early days of a group tends to rush forward over the surface;

in the group's later history to slacken speed and return on its track.* Also the groups which live the longest, returning two or more times to the visible hemisphere of the Sun, move more slowly on the whole than short-lived spots, and yield different rotation-periods during their successive apparitions.

One remarkable peculiarity is common to both the cycles, 1879–1891 and 1891–1901, and since it was brought out by Carrington's inquiry two cycles earlier it is probable that it expresses a real peculiarity of the solar rotation. In spite of the great irregularity in the rotation-periods given by the spots in any particular zone, there does appear to be a distinct tendency for the shortest mean period to be given, not at the equator, but slightly to the north of it. The curve

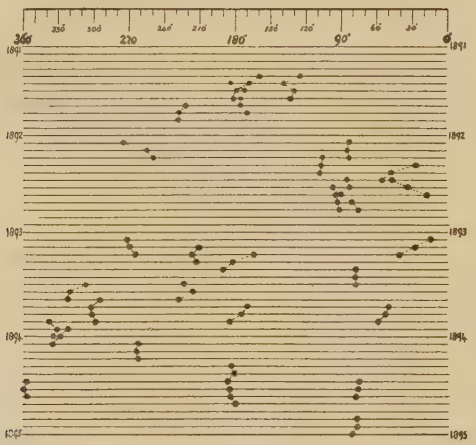

FIG. 6.—Distribution of Spot-groups of long Duration in the Years 1891 to 1894, in Heliographic Longitude.

The dots indicate the Heliographic Longitude of each group of Spots when on the Central Meridian of the Sun's Disc.

given by the different rotation-periods is not precisely symmetrical with respect to the equator, and, *on the whole*, there appears to be a tendency for the periods in the northern latitudes to lengthen more rapidly with distance from the equator than with those of the southern. A similar feature was suggested in "the Butterfly Diagram," wherein the southern hemisphere appeared to encroach slightly on the northern at the equator. The equator of rotation would appear not to coincide with the equator of figure, but to lie slightly to the north of it.†

* "Notes on some of the Spot-groups measured at the Royal Observatory, Greenwich, on Photographs of the Sun taken in the Year 1915," *Monthly Notices*, 79, 451.

† See a paper communicated to the British Astronomical Association on "Rotation Periods of the Sun as determined from Flocculi and from Sunspots" (*Journal of the British Astronomical Association*, 32, 104–107): "We thus infer a state of strain not only between the southern and northern solar hemispheres, but also between the higher and lower strata in those hemispheres," p. 106.

The different rotation periods given by long-lived spot groups during their successive apparitions are illustrated in diagram No. 6, which exhibits the distribution of longitude of all groups lasting through three or more successive rotations during the years 1891–1894. No distinction has been made in the diagram between northern and southern spots. It may be assumed that the greater the depth below the surface of the place of origin of a given spot group, the greater will be the difference of its observed rate of motion from the general surface drift.

The seventh diagram is drawn upon precisely the same plan and scale as the sixth, and shows the distribution of certain solar longitudes. But these are not the longitudes of spots on the Sun, but of the centre

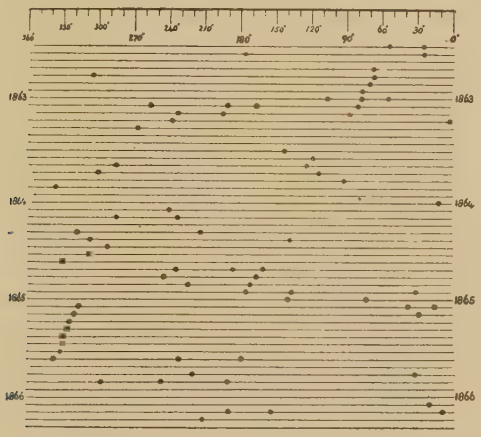

Fig. 7.—Distribution of Magnetic Storms, 1862 to 1866, for each rotation of the Sun.

Note.—The dots indicate the Heliographic Longitude of the centre of the Sun's Disc at the time of the commencement of each of the Magnetic Storms.

of the Sun's disc at the moment when a magnetic storm began to be felt upon the earth. It will be seen how similar is the character of the grouping of the associated dots in the two diagrams. In other words, the intervals between the successive returns of a group of spots to the centre of the Sun's disc as seen from the earth, are strictly comparable with the intervals between the recurrences of a magnetic storm. Magnetic storms obey the law of the Sun's rotation.

This could not be so if the influence that brings about our magnetic storms proceeded equally from every part of the solar surface and was radiated from it equally in all directions. It follows that magnetic storms are brought about by an influence which arises from restricted areas of the Sun's surface, and is discharged from such areas in restricted directions. Thus Mr. Gavin Burns has tabulated his records of auroræ (*Memoirs of the British Astronomical Association*, 23, Part I., "Report of the Section for the Observation of Auroræ and the Zodiacal Light,

1916–1919") in recent years in accordance with the solar rotation, and the tables show at a glance that the greatest displays synchronised with the return of certain special meridians to the centre of the Sun's disc; other meridians on the Sun appear to leave our atmosphere quite undisturbed. The Sun is structured, not only as between the northern and southern hemispheres, and as between different latitudes, but—as regards its emanations—as between some of its longitudes also.

There are such influences proceeding from the Sun, and the outer corona is built up in obedience to them. This has been clearly seen in the photographs which have been secured of the corona on many occasions; as, for example, in the eclipse of 1871 December 12, photographed at Baikul by Lord Lindsay's assistant, Mr. Davis, and shown in the *Memoirs of the R.A.S.*, 41, Plates 7 and 8. The chief feature of the corona of this eclipse was the massing of its greater portion into petal-shaped formations, which were called at that time "synclinal groups." In Plate 7 the synclinal group in the S.S.E., rises above a bright semicircular arc, and much fainter arches can be discerned rising above the lowest one. Above the highest of these the sides of the structure curve as if to meet; in Plate 8 they very nearly meet. Several similar synclinal groups were yet more distinctly shown in the photographs obtained by Sir W. H. M. Christie in the eclipse of 1898 at Sahdol in India, and again in the eclipse of 1901, by the members of the British expeditions to Sumatra and Mauritius.

But it was in the eclipse of 1898 that the structure of the outermost corona was photographed; and the synclinal groups, so beautifully shown on Sir W. H. M. Christie's negatives, were seen on photographs taken at Talni, with a Dallmeyer astigmatic lens of $1\frac{1}{2}$ inches aperture and 9 inches focus, to terminate in long narrow straight beams, the apex of the highest arch in each of the groups being, as it were, blown out, and driven away in a straight line into space. The longest beam was traced for at least six million miles.*

From photographs taken in these and other eclipses it would appear that over certain disturbed areas of the solar surface leaf-shaped structures are built up, from the apices of which matter seems to be driven away in long straight rod-like beams. Such beams, driven forth into space and continually replenished from beneath, would, if in the right direction, from time to time overtake the earth in its orbit, and strike it on the sunset arc. Remembering the facts brought forward by Professor Hale, that the "leader" and "trailer" of a stream in one hemisphere are of different polarities, as well as the leaders of streams in different hemispheres, we can understand how the same disturbed area may be fed with particles of different sign, which, by their mutual attraction, give rise to the rod-like beams that issue from it. These cross the space that separates the earth from the Sun, and reveal their arrival here by means of magnetic storms and auroræ.

[Thus it is evident that the disturbed state of the Sun's surface affects the earth. But there is also some evidence that the earth appears to affect the Sun's surface. This appears in a slight, but distinct, dissymmetry between the eastern and western halves of the apparent disc

* *Knowledge*, for 1898 May, pp. 107–9.

of the Sun, with respect to three orders of solar phenomena: the numbers of prominences; the areas of faculæ; and the numbers and areas of spot groups. There is some evidence that the amount and the sign of this dissymmetry vary in the course of the solar cycle. A complete explanation is still to seek.*]

These conclusions and suggestions have been derived, during the century of the history of this Society, from observations of the changes, real and apparent, of the areas of sunspots and of their distributions in longitude and latitude.

A Recent Small Sunspot in High Northern Latitude.

(*Communicated by the Astronomer Royal.*)

A very small sunspot appearing in latitude 31° N. on June 24 of this year was observed at Mount Wilson to be of negative polarity.

Whether this spot belongs to the declining or the approaching cycle is not yet certain, and the appearance of other spots in these latitudes and the determination of their magnetic polarity will, therefore, be of considerable interest with regard to the possible reversal of polarities of spots as at the last minimum.

At present there is only one photograph available at Greenwich—one taken at the Cape at $10^h\ 44^m$ G.C.T. on June 24. This gives the position of the spot as lat. 31°·5 N. and long. 35°·7. This differs from the Mount Wilson measures by about 5° in longitude, so that if the marking on the Cape photograph is confirmed, it probably represents the following component of a bipolar group, the leader of which was observed at Mount Wilson.

The following list of spots seen in solar latitudes above 30° N. and S. since the year 1917 is abstracted from the Greenwich measures of solar photographs:—

Year.	Month and Day.	Area in Millionths of Sun's Visible Hemisphere.	Latitude.	Longitude.	Longitude from Central Meridian, −East, +West.
1917	Feb. 1	6	+31°	67°	−49°
	Feb. 17	4	−30	229	−37
	Mar. 2	3	+33	134	+40
	Mar. 4–5	32	+32	106	+38 to +50
	Mar. 27	2	+51	109	−16
	Apr. 14	5	−31	181	−67

* *Monthly Notices*, **47**, 451: "An Apparent Influence of the Earth on the Numbers and Areas of Sunspots in the Cycle 1889–1901"; and *Memoirs* of the British Astronomical Association, **23**, Part II., "Report of the Section for the Observation of the Sun, 1910–1919"; "The Distribution of Faculæ between the four Quadrants of the Sun's Apparent Disc," p. 46.

The above paragraph in brackets was omitted in reading, as the allotted time had expired.

Year.	Month and Day.	Area in Millionths of Sun's Visible Hemisphere.	Latitude.	Longitude.	Longitude from Central Meridian, − East, + West.
1917	Apr. 19	3	+31°	138°	−43°
	Apr. 26	4	−30	124	+34
	Aug. 19	3	+40	34	+27
	Sept. 6	2	+32	138	+10
	Sept. 23	34	+33	350	+86
	Oct. 6	3	+31	110	+18
	Dec. 15	5	+32	245	− 5
	Dec. 26	9	−32	160	+54
1918	Jan. 12–13	15	−31	305	+64 to +76
	Jan. 29–Feb. 7	86	−32	295	−79 to +37
	Nov. 8	4	+33	236	− 6
1919	Aug. 13	3	+42	149	−23
	Dec. 12–13	22	+30	352	−24 to −10
1920			Nil*		
1921			Nil*		
1922	June 24	3	+31	36	−11

Royal Observatory, Greenwich:
1922 *September* 21.

Note by the Secretaries on a Paper by Mons. G. Shajn.

The attention of the Secretaries has been drawn to the fact that a paper by Mons. G. Shajn, of Pulkovo Observatory, reached much the same conclusions on the co-relation of the rate of melting of the Martian Polar Caps with the variation of sunspot activity as those deduced by Mons. E. M. Antoniadi in *M.N.*, **76**, 243 (1916). Mons. Shajn's paper, which is in Russian, was published in *Bulletin de la Société astronomique de Russie*, **18**, 243–259 (1912). It contains a diagram showing the curves of sunspot activity and of the rates of melting of the cap from 1877 to 1907, the two curves being nearly parallel to each other. It is well to call the attention of those interested in the subject to this paper.

* Subject to slight revision; final examination of photographs not made after 1921 July.

Photographic Observations of Comet 1922 *b* (*Skjellerup*) *made with the* 30-*in. Reflector at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

The explanations given on p. 493 apply to the following place. The parallaxes correspond to $\log \Delta = 9{\cdot}7808$, which gives $301^{s}{\cdot}0$ for light time.

	G.M.T.	Comet's R.A.	Comet's N. Decl.	Corrections for Parallax. R.A.	Corrections for Parallax. Decl.	Corrections for Aberration. R.A.	Corrections for Aberration. Decl.
1922.	h m s	h m s	° ′ ″	s	″	s	″
July 31	10 50 14·7	17 29 21·94	20 40 50·9	+0·32	+7·8	+1·03	+9·5

Royal Observatory, Greenwich:
1922 *September* 25.

Fellows will learn with deep regret of the death of the Assistant Secretary, Mr. William Henry Wesley, who passed away peacefully in his sleep on October 17, at the age of 81. He had held office for 47 years, being appointed in 1875, shortly after the Society came to Burlington House.

Erratum.

The amended key to Plate 11 published in this number is to be substituted for that published in 82, 8.

1. Seares.
2. Knobel.
3. Aitken.
4. Hertzsprung.
5. Shapley.
6. Phillips (*Secretary*).
7. Eddington (*President*).
8. Crommelin (*Secretary*).
9. St. John.
10. S. S. Hough.
11. Strömgren.
12. Fowler.
13. Hollis.
14. Olsen.
15. Dreyer.
16. Miss Cook.
17. Mrs. Dodwell.
18. Mrs. Freeman.
19. Lady Dyson.
20. Countess de Rivas.
21. Miss Cave-Browne-Cave.
22. Miss Meyer.
23. Mrs. Shapley.
24. Dodwell.
25. Lindemann.
26. Miss Herschel.
27. Miller.
28. Grove-Hills (*Treas.*).
29. Lee.
30. Lockyer.
31. Turner (*For. Sec.*).
32. Newall.
33. Maharajah of Jalawar.
34. Gwilliam.
35. Davies.
36. Dyson (*Astron. Royal*).
37. Dannreuther.
38. Rendell.
39. Cottingham.
40. Hony.
41. Byfield.
42. Maxwell.
43. Waran.
44. Heath.
45. Rowland.
46. Pendlebury.
47. Bell.
48. Baxandall.
49. Thorrowgood.
50. Burnet.
51. Milne.
52. Dingle.
53. Greaves.
54. Purey-Cust.
55. Marshall.
56. Smart.
57. Robbins.
58. Bartrum
59. Inwards.
60. Hepburn.
61. Lingham.
62. Thomson.
63. Arnold.
64. Phayre.
65. Adames.
66. Bellamy
67. Herschel.
68. Knox-Shaw.
69. Jackson.
70. Bower.
71. Allen.
72. Waterfield.
73. Witchell.
74. Levin.
75. Brewitt-Taylor.
76. Mrs. Merton.
77. Merton.
78. Ainslie.
79. Longbottom.
80. Steavenson.
81. Comrie.
82. Lenox-Conyngham.
83. Hudson.
84. Leeson.
85. Chamberlin.
86. Parr.
87. Markwick.
88. Goldsbrough.
89. Plummer.
90. Newbegin.
91. Fisher.
92. Stoney-Smith.
93. Butler.
94. Bryant.
95. Hall.

Key to Plate 11.

LIST OF ADDITIONS

TO THE LIBRARY

OF THE SOCIETY

JUNE 1921 TO JUNE 1922.

Serial Publications received in the regular course are not included in this Annual List.

An asterisk (*) indicates that the work is an excerpt.

Airy (G. B.):
Mathematical Tracts on practical Astronomy. 1st edition.
(*W. H. Wesley.*) 8vo. Cambridge, 1826

Alliata (Guilio):
Verstand contra Relativität.
(*Author.*) 8vo. Locarno, 1921

Arcetri, Reale Osservatorio:
Pubblicazioni. Fasc. 38 (1919).
(*Observatory.*) 4to. Florence, 1921

Barnard (E. E.):
——: *Sherburne Wesley Burnham.
(*Author.*) 8vo. Northfield, Minn., 1921
——: *The Variability of Nova Cygni, 1876 (Schmidt).
(*Author.*) 8vo. London, 1921
——: *Star Streams.
(*Author.*) 8vo. Toronto, 1922

Bergstrand (Ö.):
*Über die effektiven Wellenlängen der Milchstrassensterne.
(*Upsala Observatory.*) 4to. Kiel, 1921

Berlin, Astronomisches Rechen-Institut:
Astronomisches Jahrbuch, 1923.
(*Rechen-Institut.*) 8vo. Berlin, 1921

Bern, Schweizerischen Geodätischen Kommission:
Astronomisch-geodätische Arbeiten in der Schweiz.
(*Schweizerischen Naturforschenden Gesellschaft.*) 4to. Bern, 1921

Bern, Universität:
Jahrbuch der Philosophischen Fakultät. Band I., 1921.
(*University.*) 8vo. Bern, 1921

Bibliotheca Chemico-Mathematica:
Vols. 1, 2.
(*Purchased.*) 8vo. London, 1921

Bologna, Observatorio Astronomico:
Pubblicazioni. Vol. 1, Nos. 1, 2, 3.
(*Observatory.*) 8vo. Bologna, 1921

Bordeaux, Société Astronomique:
Bulletin. No. 4, Jan.–March 1921.
(*Society.*) 8vo. Bordeaux, 1921

Brahe (Tycho):
Tychonis Brahe Dani Opera Omnia: editet J. L. E. Dreyer. Tomus 3–6.
(*Editor.*) 4to. Hauniæ, 1916–1922

Breson (Elias):
Communications (No. 4) about Observations of the Planet Jupiter during Opposition, 1916–17.
(*Author.*) 4to. Helsingfors, 1921

Brussels, Académie Royale:
L'Académie Royale de Belgique depuis sa fondation.
(*Academy.*) 8vo. Brussels, 1922

Calcutta, Institution of Engineers (India):
Journal. Vol. 1.
(*Institution.*) 8vo. Calcutta, 1921

Cape of Good Hope, Royal Observatory:
—— : 2nd Fundamental Catalogue of 1846 stars made during the years 1912–1916, under the direction of S. S. Hough.
(*Observatory.*) 4to. Edinburgh, 1920
—— : Results of Meridian Observation of Stars made 1912–1917.
(*Observatory.*) 4to. Edinburgh, 1921

Christchurch (N.Z.), Magnetic Observatory:
Hourly Values of the Magnetic Declination and the Horizontal Magnetic Force for the years 1910 and 1920.
folio, Wellington, 1921

Clarke, Cumberland:
Astronomy in the Poets.
(*Author.*) 8vo. Bournemouth [1921]

Cordoba, Observatorio Nacional Argentino:
Resultados. Vol. 23: Catálogo de las Zonas de Exploración: Zonas $-27°$ à $-32°$. B.
(*Observatory.*) 4to. Cordoba, 1914

Cortie (Rev. A. L.):
*Sir Norman Lockyer.
(*Norman Lockyer Observatory.*) 8vo. Chicago, 1921

Cracow Observatory:
Circulaires. Nos. 10, 11.
(*Observatory.*) folio, Cracow, 1921

Crompton (Harold):
Nebular Zones and the Formation of Planets.
(*Author.*) 8vo. London, 1922

Dalladay (A. J.) and F. Twyan:
*The Measurement of small variations of Refractive Index throughout Meltings of Optical Glass.
(*Adam Hilger, Ltd.*) 8vo. London, 1921

Dictionary of Applied Physics: edited by Sir Richard Glazebrook.
(*Purchased.*) 4to. London, 1922

Vol. 1. Mechanics, Engineering, Heat.

Eddington (A. S.):
——: *Das Strahlungsgleichgewicht der Sterne.
(*Author.*) 8vo. Berlin, 1921
——: The Theory of Relativity and its Influence on Scientific Thought. (The Romanes Lecture.)
(*Author.*) 8vo. Oxford, 1922

Encyclopädie der Mathematischen Wissenschaften:
Band 6 (2), Heft 6, 7; Band 6 (1 B), Heft 4.
(*Purchased.*) 8vo. Leipzig, 1915–1920

Euler (Leonhard):
Opera Omnia; sub auspiciis Societatis Scientiarum Naturalium Helveticæ. Series I. Opera Mathematica.
(*Purchased.*) 4to. Lipsiæ et Berolini, 1921

Vol. 6. Commentationes Algebraicæ.

Hagen (Rev. J. G.):
Die Veränderlichen Sterne. Band I. Geschichtlich-technischer Teil. Lieferung 4.
(*Purchased.*) 4to. Freiburg, 1921

Hale (G. E.):
The New Heavens.
(*Author.*) 8vo. New York, 1922

Hamburger Sternwarte in Bergedorf:
——: Astronomische Abhandlungen. Band 2, No. 6.
(*Observatory.*) 4to. Hamburg, 1922
——: Mitteilungen. Band 5, No. 14.
(*Observatory.*) 8vo. Hamburg, 1922

Hicks (W. M.):
A Treatise on the Analysis of Spectra.
(*Purchased.*) 8vo. Cambridge, 1922

Howarth (O. J. R.):
The British Association for the Advancement of Science: A Retrospect, 1831–1921.
(*British Association.*) 8vo. London, 1922

Jarry-Desloges (R.):
Observations des Surfaces Planétaires.
Fasc. 4. Années 1913, 1914.
(*Author.*) 8vo. Abbeville, 1921

Journal of Scientific Instruments:
Preliminary number.
(*Institute of Physics.*) 8vo. London, 1922

Kalocsa, Haynald Observatory:
Publicationen. Heft xi.
(*Observatory.*) 4to. Kalocsa, 1922

Kohl (O.):
*Die Rotverschiebung der Spektrallinien der Sterne des Taurus stromes.
(*Author.*) 8vo. Göttingen, 1921

Krüger (F.):
Indexkatalog nebst Ergänzungen zu den Sternfarbenverzeichnissen von Hagen, Krüger, und Osthoff.
(*Specola Vaticana.*) 4to. Vienna, 1917

Kyoto, University Observatory:
Bulletin. Nos. 1–18.
(*Observatory.*) folio, Kyoto, 1921

Leiden, Astronomical Institutes of the Netherlands:
Bulletin. Nos. 1–17.
(*Institutes.*) 4to. Leiden, 1921–1922

Linde (J. C. van de):
De Verdeeling der heldere Sterren.
(*Utrecht Observatory.*) 8vo. Rotterdam, 1921

Lisbon, Academia des Sciencias:
Boletin da Classe de Letras. Vol. 13, fasc. 1, 2.
(*Academy.*) 8vo. Coimbra, 1920–1921

Liverpool, Literary and Philosophical Society:
Proceedings. No. 66, Sessions 1918–1921.
(*Society.*) 8vo. Liverpool, 1921

London, British Astronomical Association:
Observer's Handbook for 1922.
(*Association.*) 8vo. London, 192

London, Royal Society Catalogue of Scientific Papers:
Vols. 16, 17.
(*Purchased.*) 4to. Cambridge, 1918–1921

London, University College:
Tracts for Computers. No. 5.
(*University College.*) 8vo. Cambridge, 1921

Lourenço Marques, Observatorio Campos Rodrigues:
Relatório. Ano de 1919.
(*Observatory.*) 4to. Lourenço Marques, 1921

Lundmark (K.):
——: *Mitteilung über Nova Cygni, 1920.
(*Upsala Observatory.*) 4to. Kiel, 1921
——: *Ein neuer Veränderliche voor P Cygni-Typus.
(*Upsala Observatory.*) 4to. Kiel, 1921

Lyons Observatory:
Bulletin. Year 4, No. 4.
(*Observatory.*) 8vo. Lyons, 1922

Madison, Washburn Observatory:
Publications. Vol. 10, pt. 4.
(*Observatory.*) 4to. Madison, 1919
G. C. Comstock. Observations of Double Stars.

Masaryk University:
Publications. Nos. 1–15.
(*University.*) 8vo. Brno, 1921–1922

Mascart (Jean):
Impressions et Observations dans un Voyage à Tenerife.
(*Author.*) 8vo. Paris [*n.d.*]

Meesters (P.):
De in Nederland voor het bloote oog zichtbare Sterren. Eerste Lijst.
(*Utrecht Observatory.*) 8vo. Amsterdam, 1920

Montucla (J. E.):
Histoire des Recherches sur la Quadrature du Cercle.
(*Purchased.*) 12mo. Paris, 1754

Moscow, Observatoire Astronomique:
Annales. Série II., Vol. 6. Observations photométriques.
(*Observatory.*) 4to. Moscow, 1917

Naples, Reale Osservatorio di Capodimonte:
Memorie. No. 6.
(*Observatory.*) 4to. Naples, 1921

Newcomb Engelmann:
Populäre Astronomie. 6th edition.
(*Purchased.*) 8vo. Leipzig, 1921

Nort (H.):
The visual photometric scales of the Harvard Observatory.
(*Author.*) 8vo. Gouda, 1921

Osthoff (Heinrich):
Die Farben der Fixsterne, auf Grund eigener Beobachtungen.
(*Specola Vaticana.*) 4to. Vienna, 1916

Pannekoek (A.):
—— : *The Distance of the Dark Nebulæ in Taurus.
(*Amsterdam Astronomical Institute.*) 8vo. Amsterdam, 1920
—— : The Local Star-System.
(*Amsterdam Astronomical Institute*). 8vo. Amsterdam, 1921

Paris, Bureau International de l'Heure:
—— : Rapport Annuel sur les travaux effectués, Année 1921.
(*Bureau.*) 8vo. Paris, 1921
—— : Bulletin horaire. Tome I., No. 1.
(*Bureau.*) 8vo. Paris, 1922

Perth Observatory, Western Australia:
Astrographic Catalogue, 1900·0. Perth Section, Dec. $-31°$ to $-41°$. Vols. 5–8; 13–16. Measures of Rectangular Co-ordinates, etc.
(*Observatory.*) 4to. Perth, 1919–1921

Philip (Alexander):
The Calendar: its history, structure, and improvement.
(*Purchased.*) 8vo. Cambridge, 1921

Philippot (H.):
*L'Usage des Co-ordonnées galactiques.
(*Author.*) 12mo. Brussels, 1922

Philippot (H.), E. Delaporte, et E. Moreau:
Positions moyennes pour 1914 d'étoiles de comparaison de la comète 1913 f. (Delavan).
(*Observatoire Royal de Belgique.*) 4to. Brussels, 1922

Pickering (Wm. H.):
—— : Mars.
(*Gorham Press.*) 8vo. Boston, 1922
—— : Monthly Report on Mars. No. 24.
(*Author.*) 8vo. Northfield, Minn., 1922

Pomona College Astronomical Society:
Publications. Vol. 7, No. 1. New Series.
(*Society.*) 8vo. Claremont, Cal., 1922

Prague, Observatoire National de la République Tehécoslovaque:
—— : Magnetische und Meteorologische Beobachtungen, 1913–1916.
(*Observatory.*) 4to. Prague, 1914–1917
—— : Almanac for 1922.
(*Observatory.*) 8vo. Prague, 1922

Prague, Sternwarte der deutschen Universität:
Astronomische Beobachtungen. No. 1.
(*Observatory.*) 4to. Prague, 1921

A. Kaiser. Beobachtungen von Finsternissen.

Revue d'Optique, Thèorique et Instrumentale:
Nos. 1–4.
(*Editor.*) 8vo. Paris, 1922

Rogers (Inkerman):
The Progress of Horology.
(*Author.*) 8vo. London, 1921

Rohr (Moritz von):
Die binokularen Instrumente.
(*Purchased.*) 8vo. Berlin, 1920

San Fernando, Observatorio de Marina:
——: Astrographic Catalogue, 1900·0. San Fernando Section, Dec. −3° to −9°. Vols. 1, 2.
(*Observatory.*) 4to. San Fernando, 1921–1922
——: Carta Fotográfica del Cielo. Zonas −3°, −5°, −7°, −9° (33 charts).
(*Observatory.*)

See (T. J. J.):
*New Theory of the Æther. Fifth and sixth papers.
(*Author.*) 4to. Kiel, 1922

Scheiner-Graff:
Astrophysik. (Dritte völlig neubearbeitete Auflage von J. Scheiner "Populäre Astrophysik," von Prof. Dr. K. Graff.)
(*Purchased.*) 8vo. Berlin, 1922

Schönberg (E.):
Photometrische Untersuchungen über Jupiter und das Saturnsystem.
(*Author.*) 8vo. Helsingfors, 1921

Skey (H. F.):
——: *Some Results from the preliminary discussion of the Annual Variation of Horizontal Magnetic Force at Christchurch.
(*Hector Observatory.*) 8vo. Wellington, N.Z., 1921
——: *The recent Magnetic Storm, 14th–17th May 1921.
(*Hector Observatory.*) 8vo. Wellington, N.Z., 1921

Souleyre (A.):
La Structure des Planètes.
(*Purchased.*) 8vo. Paris, 1917

Stockholm, K. Svenska Vetenskaps-Akademie:
Lefnadsteckningar. Band 5, Häfte 2.
(*Academy.*) 8vo. Upsala, 1915–1920

Sydney Observatory, N.S.W.:
Catalogue of 1068 "Intermediate" Stars, situated between 51° and 65° South declination, for the equinox 1900, from observations made during the years 1918–1919.
(*Observatory.*) 4to. Sydney, 1921

Tacubaya, Observatorio Astronómico Nacional:
Carta fotografica del Cielo. Zonas −11°, −13°, −15° (10 charts).
(*Observatory.*)

Tannery (Paul):
Mémoires Scientifiques: III. Sciences Exactes dans l'Antiquité, 1889–1913.
(*Purchased.*) 8vo. Paris, 1915

Teramo, R. Specola di Collurania:
Pubblicazioni. Serie A, Vol. 1; Serie B, Vol. 1.
(*G. Zappa.*) 4to and 8vo. Rome, 1921

Turin, R. Osservatorio Astronomico di Pino Torinese:
Pubblicazioni. No. 1, Anno 1921.
(*Observatory.*) 8vo. Turin, 1921

Uccle, Institut Royal Météorologique:
Annales. Tome I., fasc. 1. Années 1914, 1915.
(*Institute.*) 4to. Brussels, 1919

Vanderlinden (H. L.):
*De Kracht van Coriolis in Einstein's gravitatie theorie.
(*Author.*) 8vo. Uccle, 1922

Viaro (B.):
——: *Il Pianetino (8) Flora.
(*Padua-Observatory.*) 8vo. Venice, 1921
——: *Osservazioni fotometriche della Variabile a corto periodo RT Aurigæ.
(*Padua Observatory.*) 8vo. Padua, 1921

Vienna, Akademie der Wissenschaften:
Sitzungsberichte, Math.-Nat. Classe. Band 130, Heft 3, 4.
(*Academy.*) 8vo. Vienna, 1921

Washington, National Academy of Sciences:
National Research Council Bulletin. Vol. I., pt. 3, No. 11.
(*Academy.*) 8vo. Washington, 1921

Waters (H. H.):
Astronomical Photography for Amateurs.
(*Purchased.*) 8vo. London, 1921

Wellington, N.Z., Hector Observatory:
Circular. Nos. 1–6.
(*Observatory.*) folio, Wellington, 1921

Woltjer (J., Jun.):
——: *The longitude of Hyperion's pericentre and the mass of Titan.
(*Leiden Observatory.*) 8vo. Amsterdam, 1918
——: *On the perturbations in the motion of Hyperion proportional to the first power of Titan's eccentricity.
(*Leiden Observatory.*) 8vo. Amsterdam, 1919

Zappa (Giovanni):
——: *Per una verifiqa sperimentale della teoria di Relatività di Einstein.
(*Author.*) 4to. Rome, 1917
——: *Per la determinazioni dei moti propri delle Stelle di Santini.
(*Author.*) 8vo. Venice, 1917

Zeipel (H. v.):
——: *L'Œuvre astronomiques d'Henri Poincaré.
(*Upsala Observatory.*) 4to. Stockholm, 1920
——: Recherches sur le mouvement des petites planètes.
(*Upsala Observatory.*) 8vo. Upsala, 1921
——: *Die Bestimmung der Massen der Sterne aus ihrer Verteilung in den Sternhaufen.
(*Upsala Observatory.*) 4to. Kiel, 1921

Zeipel (H. v.) and J. Lindgren:
*Photometrische Untersuchungen der Sterngruppe Messier 37 (N.G.C. 2099).
(*Upsala Observatory.*) 4to. Stockholm, 1921

PHOTOGRAPHS PRESENTED TO THE SOCIETY.

Cordoba, Observatorio Nacional Argentino. Photographs of Nebulæ (six slides).
(*Prof. C. D. Perrine.*)

Professor J. Evershed. Three slides showing widened lines in the Spectrum of Sirius.

Mount Wilson Observatory. Photographic map of the Sun-spot Spectrum, and two transparencies of Sun-spots.
(*Prof. G. E. Hale.*)

INDEX

PAGE

Address to the King, on the occasion of the Society's centenary celebration..... 432
Annual General Meeting, Report of the proceedings of the..... 233
Argelander's method, Note on the accidental errors of, P. M. Ryves..... 78
Assets and present property of the Society..... 242
Associates' addresses delivered on the occasion of the Society's centenary celebration..... 439
Asteroids, On long-period inequalities in the movements of, W. M. H. Greaves..... 149
Astrographic Catalogue, The number of stars of different magnitudes in the Hyderabad, T. P. Bhaskaran..... 183
Atkinson, R. d'E., Note on the pressure in the reversing layer in stars..... 396
Auditors of the Treasurer's accounts for 1921, Appointment of..... 62
——————————————— Report of the..... 238

Baillaud, B., Note sur la précision des signaux horaires émis de 1910 au 31 décembre 1919 par le Bureau International de l'heure à Paris..... 491
Balfour, Lord, Address at the centenary dinner..... 442
Barnard, E. E., The variability of Nova Cygni, 1876 (Schmidt)..... 116
——————— On the light changes and other peculiarities of Nova Persei No. 2 (Anderson 1901)..... 457
Bhaskaran, T. P., The number of stars of different magnitudes in the Hyderabad Astrographic Catalogue. Fourth paper..... 183
Bilt, J. van der, Observations of minor planets made with the 10-inch refractor of the University Observatory, Utrecht, Holland..... 84
Bowyer, W., Sir F. W. Dyson and, Reception of W/T signals at the Royal Observatory, Greenwich..... 193
Brown, A. N., Observations of (Ch. 7085) RT Cygni in 1917–22..... 405
Bruce medal, Presentation of, to Sir Frank Dyson..... 439
Butler, C. P., Systematic distribution of solar calcium flocculi. I. Inclination of elongated groups..... 334

Candidates proposed..... 1, 61, 121, 234, 291, 321, 391, 446
Celestial photographs..... 243
Centenary celebration..... 431–442
Chapman, S., Diffusion and viscosity in giant stars..... 292
Comets:—
Observations of comet 1921 *a* (Reid) made at the University Observatory, Rostov-on-Don, S. D. Tscherny..... 59
Council note on the discovery of, in 1921..... 268
Note on comet 1906 VI. (Metcalf), G. Merton..... 381
Approach of Brooks's periodic comet to Jupiter, C. Pokrovsky..... 383
Determination of the elements of the orbit of a planet or a comet by means of the variation of two geocentric distances, M. Subbotin..... 383
Photographic observations of comet 1922 *b* (Skjellerup) made with the 30-in. reflector, Royal Observatory, Greenwich..... 493, 545

PAGE

Comrie, L. J., New Zealand observations during the lunar eclipse of 1921 April 21 303
———————— Stars occulted during lunar eclipses in 1921 304
Conversazione, Proceedings at 431
Cook, Miss A. G., and J. P. M. Prentice, Observations of the meteors of the ε Arietid Radiant in 1921 309
Correction to the Catalogue of William Herschel papers 232
Cortie, Rev. A. L., Terrestrial magnetic disturbances and sun-spots 170

Davidson, C. R., Parallax of B.D. 80°694 180
———————— and E. Martin, Effective wave-lengths of stars in relation to spectral type and colour-index 65
Denning, W. F., A detonating fireball in sunshine 305
———————— Meteoric phenomena, February 7-22 307
Dinner held at the Criterion Restaurant on the occasion of the Society's centenary celebration 442
Doig, Peter, The spectra of physically connected pairs and the giant and dwarf theory 372
———————— Note on the parallaxes of open and moving clusters 461
Dreyer, J. L. E., Address on the history of the Society, delivered on the occasion of the centenary celebration 438
Dyson, Sir Frank, Star-streaming in the region round the North Pole 19
———————— and W. Bowyer, Reception of W/T signals at the Royal Observatory, Greenwich 193

Eddington, A. S., Address delivered by, on the award of the Gold Medal to Dr. James Hopwood Jeans 279
———————— Introductory address delivered on the occasion of the Society's centenary celebration 432
Einstein relativity theory of gravitational effect, Note on the path of a ray of light in the, A. R. Forsyth 2
Errata 494, 545
Espin, Rev. T. E., Micrometrical measures of double stars (16th series) 187
Evershed, J., Widened lines in the spectrum of Sirius 392

Fellows elected 1, 61, 121, 234, 291, 321, 391, 445
Forbes, George, Solar motion from 1922 radial velocities 174
Forsyth, A. R., Note on the path of a ray of light in the Einstein relativity theory of gravitational effect 2

Gifts to the general funds of the Society 240
Gill, Sir David, Bequest 242
Greaves, W. M. H., The existence theorem for certain periodic orbits in the restricted problem of three bodies 139
———————— On long-period inequalities in the movements of asteroids whose mean motions are nearly half that of Mars 149
———————— Note on Dr. Goldsbrough's paper on the influence of satellites upon the form of Saturn's ring 356
———————— On the behaviour of a small body within the Cassini division of Saturn's ring 360
Greenwich, Royal Observatory, Stellar parallaxes 34
———————— Mean areas and heliographic latitudes of sun-spots in the year 1917 62
———————— Variation of latitude at Greenwich in 1921 297
———————— Mean areas and heliographic latitudes of sun-spots in the year 1918 483
———————— Photographic observations of comet 1922 *b* (Skjellerup) made with the 30-in. reflector 493, 545
———————— A recent small sun-spot in high northern latitude 543

PAGE
Hale, G. E., Invisible sun-spots ... 168
Halm, J., On a method of determining photographic star magnitudes without the use of screens or gratings ... 472
——— On the rotation of the sun's reversing layer ... 479
Herschel papers, Correction to the Catalogue of ... 232

Interferometer measurements, The theory of astronomical, H Spencer Jones ... 513

Jeans, J. H., The motions of stars in a Kapteyn-universe ... 122
——— The dynamics of moving clusters ... 132
Jones, H. Spencer, A determination of the photographic magnitude scale of the North Polar sequence ... 21
——— The theory of astronomical interferometer measurements ... 513
Jordan, Frank C., Note on the question of secondary oscillations in the light curves of Cepheid variables ... 38
Jupiter, Eclipses of Jupiter's satellites I. and II. in 1921, Sydney Observatory ... 58
——— The third satellite of, Rev. T. E. R. Phillips ... 96
——— On the tawny hue of Jupiter's equatorial zone, A. Stanley Williams ... 417
——— Recent Jupiter observations, Frank Sargent ... 418

Kepler's equation, The slide-rule as an auxiliary to the solution of, A. E. Levin 177

Lambert's theorem, On the application of, to the determination of orbits, M. T. Subbotin ... 419
Latitude, Variation of, in 1921, Royal Observatory, Greenwich ... 297
Levin, A. E., The slide-rule as an auxiliary to the solution of Kepler's equation 177
Library, List of donors to the ... 278
——— List of additions to the ... [181]
Lockyer, W. J. S., The use of a graduated wedge in stellar classification and parallax work ... 226
——— On the relationship between solar prominences and the corona 323
Lundmark, K., and W. J. Luyten, On the determination of the colour-equivalent of a star, with special reference to the effective wave-length and its relation to spectral class ... 495
Luyten, W. J., Note on the actual determination of photographic and photo-visual magnitudes ... 510
——— K. Lundmark and, On the determination of the colour-equivalent of a star, with special reference to the effective wave-length and its relation to spectral class ... 495

Martin, E., C. Davidson and, Effective wave-lengths of stars in relation to spectral type and colour-index ... 65
Martin, L. C., Star discs. (3), The effect of coma ... 310
Maunder, E. W., The sun and the sun-spots, 1820–1920 ... 534
Mercury, On the motion of the perihelion of, W. M. Smart ... 12
Merton, G., Note on comet 1906 VI. (Metcalf) ... 381
——— and W. H. Steavenson, Occultations of stars observed during the lunar eclipse of 1921 October 16 ... 303
Meteors, A detonating fireball in sunshine, W. F. Denning ... 305
——— Meteoric phenomena, February 7–22, W. F. Denning ... 307
——— Observations of the meteors of the ϵ Arietid Radiant in 1921, Miss A. G. Cook and J. P. M. Prentice ... 309
Milne, E. A., The temperature in the outer atmosphere of a star ... 368
Minor Planets, Observations of, J. van der Bilt ... 84
——— Council note on the discovery of, in 1921 ... 267
Moon, Occultations observed during the lunar eclipse of 1921 October 16 ... 57
Moreau, F., H. Philippot and, Note sur la détermination de l'heure ... 201

Nebulæ, Note on four interesting, C. D. Perrine ... 486
——— The planes of the spiral, in relation to the line of sight, J. H. Reynolds 510
Newall, H. F., On the law of rotation of the sun ... 101

PAGE

Newall, H. F., On a proposal of a new method of securing spectroscopic observations of the rotation of the sun 115
Newbegin, A. M., Results of solar prominence observations 1921 330
Newton, H. W., Centres of sun-spot activity during the year 1917 171

Obituary Notices: Associates:—
Burnham, Sherburne Wesley (*Fellow and Associate*) 258
Foerster, Wilhelm 263
Nyrén, Magnus 264
Obituary Notices: Fellows:—
Acocks, Herbert Francis 244
Bennett, Thomas Lawrence 319
Barnett, William 245
Bourget, Henry 246
Brooks, William R. 246
Bruford, George 247
Carpmael, Ernest 248
Clifton, Robert Bellamy 248
Drummond, Archibald Spencer 248
Gerrans, Henry Tresawna 249
Harris, Alfred Newton 249
Herschel, John 250
Lees, Edward Herbert 251
Moulton, Lord 252
Post, Charles Alfred 252
Rolston, William Edward 253
Shackleton, William 255
Spence, John 256
Spitta, Edmund Johnson 256
Taylor, Abraham Barsley 258
Williams, Harry Samuel 258
Observatoire Royal de Belgique, Note sur la réception des signaux horaires scientifiques de Paris 210
Occultation of Venus, 1921 July 1, Summary of reports on 55
Occultations observed during the lunar eclipse of 1921 October 16 57
Officers and Council, List of, 1922–23 289
Optics: Star discs. (3), the effect of coma, L. C. Martin 310

Parsons, Sir Charles, Resolution of thanks to 323
Perrine, C. D., Notes on four interesting nebulæ 486
Philippot, H., and F. Moreau, Note sur la détermination de l'heure 201
Phillips, Rev. T. E. R., The third satellite of Jupiter 96
Photometry: On a method of determining photographic star magnitudes without the use of screens or gratings, J. Halm 472
Plaskett, J. S., A very massive star 447
Plummer, H. C., On Mr. Jordan's note 42
———— On the curvature of the lines in the prismatic spectrum 466
Pokrovsky, C., Approach of Brooks's periodic comet to Jupiter 383
Prentice, Edward, Notes on two long-period variables, RT Cygni and U Persei 411
Prentice, J. P. M., Miss A. G. Cook and, Observations of the meteors of the ε Arietid Radiant in 1921 309
Presents announced 2, 62, 122, 292, 323, 392, 447
Progress and present state of the Society 235
Publications of the Society 243
Pulkovo Observatory, Wireless time service 199

Report of the Council to the hundred and second Annual General Meeting of the Society 235
Restricted problem of three bodies, The existence theorem for certain periodic orbits in the, W. M. H. Greaves 139
Reynolds, J. H., The planes of the spiral nebulæ in relation to the line of sight 510

PAGE
Russell, H. N., and D. L. Webster, Note on the masses of the stars..... 181
Ryves, P. M., Note on the accidental errors of Argelander's method..... 78
——————— Observations of Nova Cygni (No. 3)..... 299

Sampson, R. A., On the determination of time at different observatories..... 215
Sargent, Frank, Recent Jupiter observations..... 418
Saturn, 1920–1921, Council note on..... 275
——— Note on Dr. Goldsbrough's paper on the influence of satellites upon the form of Saturn's ring, W. M. H. Greaves..... 356
——— On the behaviour of a small body within the Cassini division of Saturn's ring, W. M. H. Greaves..... 360
Shajn, J., Note by the secretaries on a paper by..... 544
Smart, W. M., On the motion of the perihelion of Mercury..... 12
Spectroscopy, Astronomical, The use of a graduated wedge in stellar classification and parallax work, W. J. S. Lockyer..... 226
——————— On the curvature of the lines in the prismatic spectrum, H. C. Plummer..... 466
Star Clusters: The dynamics of moving clusters, J. H. Jeans..... 132
——————— Note on the parallaxes of open and moving clusters, Peter Doig 461
Stars, Constitution and Evolution of: Note on the masses of the stars, H. N. Russell and D. L. Webster..... 181
——————— Diffusion and viscosity in giant stars, S. Chapman..... 292
——————— The temperature in the outer atmosphere of a star, E. A. Milne..... 368
——————— Note on the pressure in the reversing layer in stars, R. d'E. Atkinson..... 396
Stars, Distribution and Motions of: Star-streaming in the region round the North Pole, Sir Frank Dyson..... 19
——————— The motions of stars in a Kapteyn-universe, J. H. Jeans..... 122
——————— The dynamics of moving clusters J. H. Jeans..... 132
Stars, Double and Multiple: Micrometrical measures of double stars (16th series), Rev. T. E. Espin..... 187
——————— A very massive star, J. S. Plaskett..... 447
Stars, Magnitudes of: A determination of the photographic magnitude scale of the North Polar sequence, H. Spencer Jones..... 21
——————— On a method of determining photographic star magnitudes without the use of screens or gratings, J. Halm..... 472
——————— Note on the actual determination of photographic and photo-visual magnitudes, W. J. Luyten..... 510
Stars, New:—
Nova Cygni III., 1920, The spectrum of F. J. M. Stratton..... 44
Nova Cygni, 1876 (Schmidt) The variability of, E. E. Barnard..... 116
Nova Cygni III., Observations of, P. M. Ryves..... 299
Nova Persei (1901) Observations of, 1921–1922, W. H. Steavenson..... 403
——————— On the light changes and other peculiarities of, E. E. Barnard..... 457
Novæ, Recent observations of, W. H. Steavenson..... 49
Stars, Occultations of: Occultations of stars observed during the lunar eclipse of 1921 October 16, G. Merton and W. H. Steavenson..... 303
——————— New Zealand observations during the lunar eclipse of 1921 April 21, L. J. Comrie..... 303
——————— Stars occulted during lunar eclipses in 1921, L. J. Comrie..... 304
Stars, Parallaxes of: Stellar parallaxes, Royal Observatory, Greenwich..... 34
——————— Parallax of B.D. 80°694, C. R. Davidson..... 180
Stars, Spectra of: Effective wave-lengths of stars in relation to spectral type and colour-index, C. Davidson and E. Martin..... 65
——————— Council note on stellar spectroscopy in 1921..... 272

PAGE

Stars, Spectra of: The spectra of physically connected pairs and the giant and dwarf theory, Peter Doig 372
—— ——— Widened lines in the spectrum of Sirius, J. Evershed 392
——— ——— On the determination of the colour-equivalent of a star, with special reference to the effective wave-length and its relation to spectral class, K. Lundmark and W. J. Luyten 495
Stars, Variable: Note on the question of secondary oscillations in the light curves of Cepheid variables, Frank C. Jordan 38
——— ——— A new variable star in Cygnus, A. Stanley Williams 300
——— ——— Observations of (Ch. 7085) RT Cygni in 1917–22, A. N. Brown 405
——— ——— Notes on two long-period variables, RT Cygni and U Persei, Edward Prentice 411
——— ——— Two variable stars in Gemini, A. Stanley Williams 416
——— ——— A probably variable star in Gemini, A. Stanley Williams 416
Steavenson, W. H., Recent observations of Novæ 49
——— ——— Observations of Nova Persei (1901), 1921–1922 403
——— ——— G. Merton and, Occultations of stars observed during the lunar eclipse of 1921 October 16 303
Stratton, F. J. M., The spectrum of Nova Cygni III., 1920, ... 44
Subbotin, M., Determination of the elements of the orbit of a planet or a comet by means of the variation of two geocentric distances 383
——— ——— On the application of Lambert's theorem to the determination of orbits 419
Sun, Chromosphere and Prominences: On the relationship between solar prominences and the corona, W. J. S. Lockyer 323
——— ——— Results of solar prominence observations, 1921, A. M. Newbegin 330
Sun, Eclipses of the: Summary of observations of the solar eclipse of 1921 April 7 54
——— ——— Observations made during the partial eclipse of the sun on 1922 March 28 at the University Observatory, Rostov-on-Don, S. D. Tscherny 430
Sun, Spectrum of, etc.: On a proposal of a new method of securing spectroscopic observations of the rotation of the sun, H. F. Newall 115
——— ——— Solar motion from 1922 radial velocities, George Forbes 174
——— ——— Council note on solar research in 1921 270
Sun-spots and Faculæ: Mean areas and heliographic latitudes of sun-spots in the year 1917, Royal Observatory, Greenwich 62
——— ——— On the law of rotation of the sun, H. F. Newall 101
——— ——— Invisible sun-spots, G. E. Hale 168
——— ——— Terrestrial magnetic disturbances and sun-spots, Rev. A. L. Cortie 170
——— ——— Centres of sun-spot activity during the year 1917, H. W. Newton 171
——— ——— Council note on solar activity in 1921 269
——— ——— Systematic distribution of solar calcium flocculi. I. Inclination of elongated groups, C. P. Butler 334
——— ——— On the rotation of the sun's reversing layer, J. Halm. 479
——— ——— Mean areas and heliographic latitudes of sun-spots in the year 1918, Royal Observatory, Greenwich 483
——— ——— The sun and sun-spots, 1820–1920, E. W. Maunder ... 534
——— ——— A recent small sun-spot in high northern latitude, Royal Observatory, Greenwich 543
Sydney Observatory, Eclipses of Jupiter's satellites I. and II. in 1921 58

Time: Note sur la détermination de l'heure, H. Philippot and F. Moreau 201
——— On the determination of time at different observatories, R. A. Sampson 215
Treasurer's account for 1921 236
——— ——— death, announcement of 495

PAGE
Trust funds..... 241
Tscherny, S. D., Observations of comet 1921 *a* (Reid) made at the University Observatory, Rostov-on-Don..... 59
——————— Observations made during the partial eclipse of the sun on 1922 March 28 at the University Observatory, Rostov-on-Don..... 430
Turner, H. H., Biographical address delivered on the occasion of the Society's centenary celebration..... 438

Variation of two geocentric distances, Determination of the elements of the orbit of a planet or a comet by means of the, M. Subbotin..... 383
Venus, Summary on reports on the occultation of, 1921 July 1..... 55

Webster, D. L., H. N. Russell and, Note on the masses of the stars..... 181
Wesley, W. H. (Assistant Secretary), Death of..... 545
Williams, A. Stanley, A new variable star in Cygnus..... 300
——————— Two variable stars in Gemini..... 416
——————— A probably variable star in Gemini..... 416
——————— On the tawny hue of Jupiter's equatorial zone..... 417
Wireless Telegraphy: Reception of W/T signals at the Royal Observatory, Greenwich, Sir F. W. Dyson and W. Bowyer..... 193
——————— Wireless time service, Pulkovo Observatory..... 199
——————— Note sur la détermination de l'heure, H. Philippot and F. Moreau..... 201
——————— Note sur la réception des signaux horaires scientifiques de Paris, Observatoire Royal de Belgique..... 210
——————— On the determination of time at different observatories, R. A. Sampson..... 215
——————— Note sur la précision des signaux horaires émis de 1910 au 31 décembre 1919 par le Bureau International de l'heure à Paris, B. Baillaud..... 491

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXII. NO. 1. NOVEMBER 1921.

PUBLISHED BY THE ROYAL ASTRONOMICAL SOCIETY,
BURLINGTON HOUSE, LONDON, W. 1,

AND SOLD BY

WHELDON & WESLEY, LTD.
(INCORPORATING WILLIAM WESLEY & SON)
28 ESSEX STREET, STRAND, LONDON, W.C. 2.

PRICE TO NON-FELLOWS, 4*s.* 0*d.*

ANNOUNCEMENT OF PAPERS RECEIVED.

Post-cards, giving the titles of such Papers as have been received or promised up to the Monday preceding the day of the Ordinary Meeting, will be sent to those Fellows who inform the Assistant Secretary of their wish to receive them.

NOTICE TO AUTHORS.

Diagrams or other illustrations accompanying Papers should be sent in a state suitable for direct photographic reproduction. It is preferable that diagrams should be drawn on a larger scale than required, in Indian ink on white paper, all lines, etc., being sufficiently bold to bear the reduction required.

Any lettering should be written in pencil, that it may be re-drawn to the proper scale.

CONTENTS.

PAGE

Fellows elected 1

Candidates proposed . . . . . *ib.*

Presents announced 2

A. R. Forsyth, Note on the Path of a Ray of Light in the Einstein Relativity Theory of Gravitational Effect . . . . . *ib.*

W. M. Smart, On the Motion of the Perihelion of Mercury . . . 12

Sir Frank Dyson, Star-streaming in the Region round the North Pole . 19

H. Spencer Jones, A Determination of the Photographic Magnitude Scale of the North Polar Sequence 21

Royal Observatory, Greenwich, Stellar Parallaxes 34

Frank C. Jordan, Note on the Question of Secondary Oscillations in the Light Curves of Cepheid Variables 38

H. C. Plummer, On Mr. Jordan's Note 42

F. J. M. Stratton, The Spectrum of Nova Cygni III., 1920 . . . 44

W. H. Steavenson, Recent Observations of Novæ 49

The Solar Eclipse of 1921 April 7, Summary of Observations . . 54

Summary of Reports on the Occultation of Venus, 1921 July 1 . . 55

Occultations observed during the Lunar Eclipse of 1921 October 16 . 57

Sydney Observatory, Eclipses of Jupiter's Satellites I. and II. in 1921 . 58

S. D. Tscherny, Observations of Comet 1921 *a* (Reid) made at the University Observatory, Rostov-on-Don 59

Printed in Great Britain by NEILL & Co., LTD., Edinburgh, January 1922.